VOLUME I
PERFORMANCE FLIGHT TESTING PHASE

CHAPTER 2
SUBSONIC AERODYNAMICS

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PREFACE

This chapter provides you a set of subsonic aerodynamic notes. In essence, they are a "copy" of my lecture notes. As you will quickly see, they follow your text, Introduction to Flight (3rd Edition), by John D. Anderson, Jr., very closely. The text should be your primary source of study--these notes simply highlight key points and are intended to help guide you through the material.

The chapter is organized by lesson number. For example, Section 2.4 will be covered during the 4th lesson.

I'm looking forward to meeting you. I hope you will let me know what I can do to improve this chapter. These notes are for YOU. They do not "stand alone"; therefore, please do not release them indiscriminately.

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Associate Professor (USAFA)

TABLE OF CONTENTS

2.1 FUNDAMENTAL THOUGHTS AND INTRO TO THE STANDARD ATMOSPHERE

2.1.1 Fundamental Physical Quantities of a Flowing Gas

Pressure, density, temperature, and velocity--these are the key physical properties of aerodynamics. The goal of the aerodynamicist is to quantify these values at every point in the flowfield. So, let's begin by clearly defining each property...

PRESSURE "is the normal force per unit area exerted on a surface due to the time rate of change of momentum of the gas molecules impacting on that surface". In your classroom (Edwards AFB on a "standard day"), the atmospheric pressure is roughly 1932 psf (pounds per square foot). Air molecules, moving about in "space", carry with them their momentum (mass times velocity). When the molecules strike a surface (e.g. the chalkboard), there is an exchange of momentum, and thus a force (from Newton's Second Law) is exerted on the surface--this is the "atmospheric pressure".

Dimensions of pressure (p) are: Force per unit area
Units of pressure are: (EE) psf (pounds per square ft)
                                     (SI) Newtons per square meter

The DENSITY of air is the mass of air per unit volume. Dimensions of density (ρ) are: Mass per unit volume Units of density are: (EE) slugs per cubic foot
                                     (SI) kg per cubic meter

TEMPERATURE is a measure of the average kinetic energy of the air particles--a high temperature flow implies fluid particles moving randomly at high speeds. In aerodynamics, the absolute temperature scale is typically used. The following are the conversions from degrees Fahrenheit and Celsius to Rankine and Kelvin, respectively:

$$°F + 460 = °R$$
$$°C + 273 = K$$

VELOCITY is a vector quantity--it has both magnitude and direction. The velocity at any fixed point in the flowfield is the velocity of an infinitesimally small fluid element as it sweeps through that point.

Dimensions of velocity   ($\vec{V}$)   are: Length per unit time

Units of velocity are: (EE) feet per sec (fps)
                                     (SI) meters per sec

These four properties are so-called "point properties". In general, they vary from point to point within the flowfield. Additionally, these properties can be a function of time--this is termed "unsteady" flow. Frequently, the flow is assumed to be "steady"--this removes the time dependency from the analysis.

Another important concept, which relates to velocity, is the definition of a "streamline". A streamline is a curve which is tangent to the velocity vectors in a flow. For example, refer to the following figure:
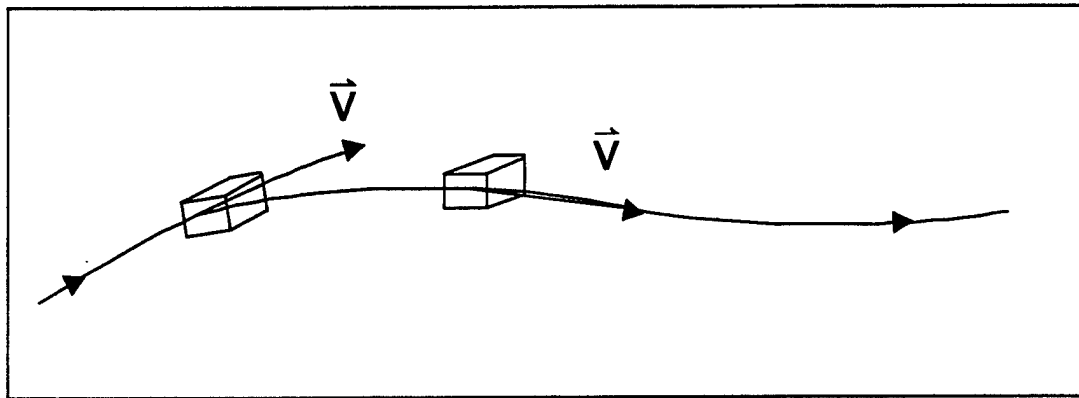
FIGURE 2.1   A TYPICAL STREAMLINE

Consider two arbitrary streamlines in the flow, as shown below. A consequence of the definition of a streamline is that the mass flow (so many kg/s) passing through station 1 must be the same as that passing through station 2. Shortly, the significance of this observation will become clear.
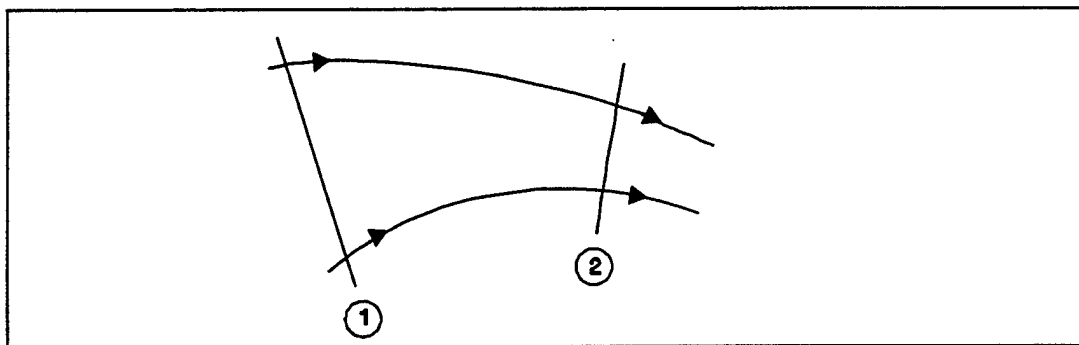
FIGURE 2.2   ARBITRARY STREAMLINES OF THE FLOW

2.1.2 The Source of all Aerodynamic Forces

The only way nature can communicate an aerodynamic force on a solid object is through pressure and shear stress distributions. As stated earlier, pressure always acts perpendicular to the surface. Shear stress (also force per unit area), on the other hand, acts tangentially to the surface. As shown below, the net effect is an aerodynamic force.
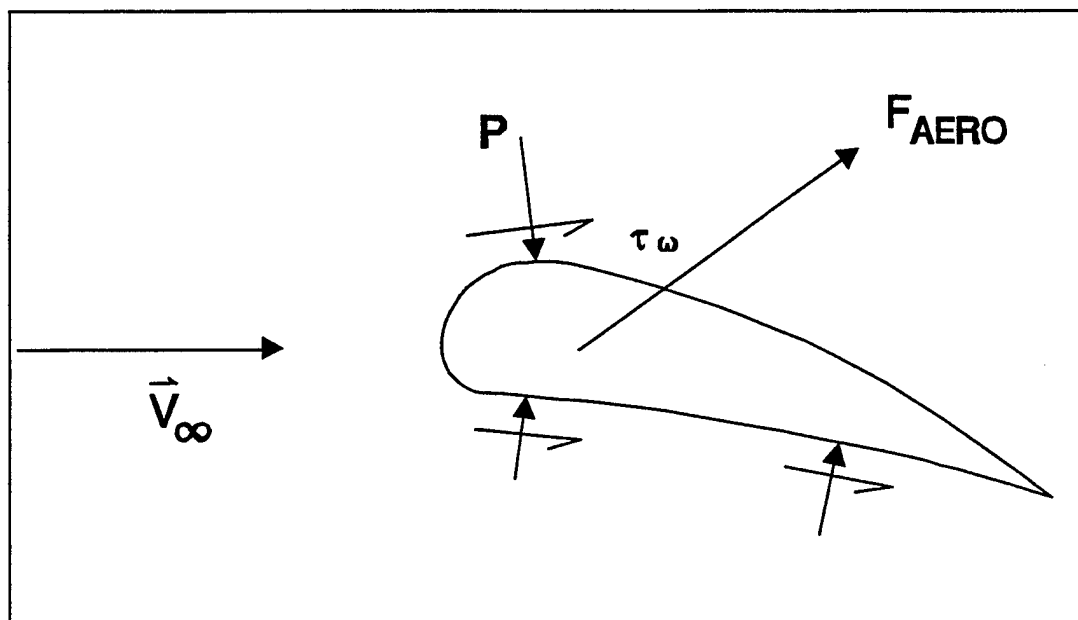
FIGURE 2.3   SOURCES OF AN AERODYNAMIC FORCE

2.1.3 Equation of State for a Perfect Gas

A perfect gas is one in which intermolecular forces are assumed to be negligible. The equation governing a perfect gas is the following:

$$p = \rho \, RT$$

In this equation, R is the "specific gas constant", a function of the gas considered--for example, its value for air is different than that of argon. For normal air, the values are given below:

$$R = 287 \frac{J}{(kg)(K)} = 1716 \frac{ft.lb}{(slug)(^\circ R)}$$

2.1.4 Discussion on Units

In aerodynamics, a "consistent" system of units is used. In both cases (English Engineering and Systems International), the units are derived directly from Newton's Second Law, F = ma. 1 lb is the amount of force necessary to accelerate 1 slug by 1 foot per second squared, or:

$$1 \; lb = 1 \; slug \; \frac{1 \; ft}{\sec^2}$$

1 Newton is the amount of force necessary to accelerate 1 kg by 1 meter per second squared, or:

$$1 \; N = 1 \; kg \; \frac{1m}{\sec^2}$$

2.1.5 Relative Motion (not covered in Anderson)

Aerodynamic forces are dependent on the relative velocity between the aircraft and the surrounding atmosphere. Consider the following--an airfoil (cross section of a wing) on a test stand with air blown over it at 100 m/s and an airfoil "flying" at 100 m/s through still air:
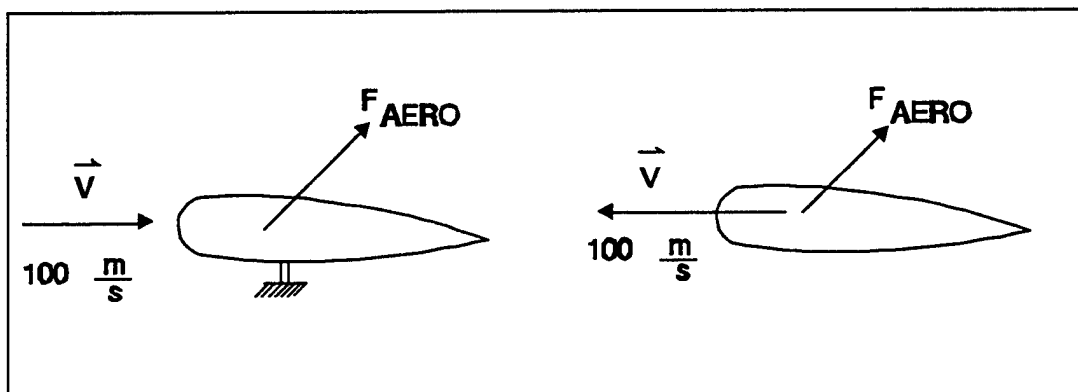
FIGURE 2.4   RELATIVE MOTION

The aerodynamic force on the two airfoils are identical--this is why wind tunnels work!

2.1.6 Introduction to the Standard Atmosphere

The "standard atmosphere" (refer to the tables in Anderson) provides a common reference for you TPS students. Level acceleration data, for example, is referenced to a "standard day"--therefore, two aircraft can be compared fairly.

The standard atmosphere tables were generated by assuming (through research) a reasonable temperature distribution with altitude. Then, by applying physical laws of nature (equation of state and the hydrostatic equation), pressures and densities were determined as a function of altitude. Frequently, it is convenient to use the following ratios--each relates the ambient property value to the sea level standard property value:

$$\delta = \frac{P}{P_{SL}} = pressure\ ratio$$

$$\theta = \frac{T}{T_{SL}} = temperature\ ratio$$

$$\sigma = \frac{\rho}{\rho_{SL}} = density\ ratio$$

NOTE: $\delta = \sigma\theta$ (from the equation of state)

2.1.7 Pressure, Temperature, and Density Altitudes

Pilots frequently use, for example, the term "pressure altitude". This is not a geometric altitude. Rather, it is the atmospheric pressure corresponding to a standard day altitude of, for example, 10,000 feet. Temperature and density altitudes are defined similarly.

2.2 BASIC AERODYNAMICS I (GOVERNING EQUATIONS, SPEED OF SOUND, MACH NUMBER, AND AERODYNAMIC FLIGHT REGIMES)

2.2.1 The Continuity Equation

The laws of aerodynamics are governed by physical principles--when applied to an appropriate "model", useful equations can be derived. The continuity equation is

based on the physical law that mass is conserved. Consider the following "streamtube" (a bundle of streamlines):
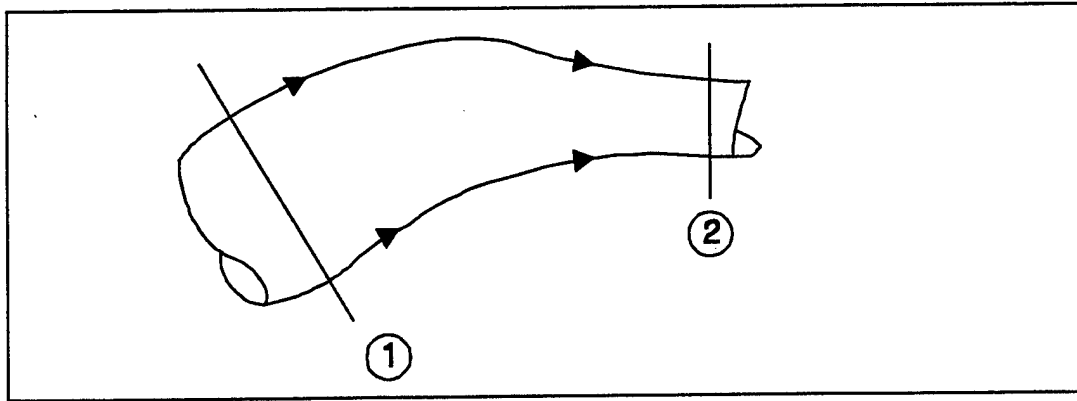
FIGURE 2.5   A STREAMTUBE

Assume that the flow is steady--properties everywhere in the flowfield are time independent. A consequence of this assumption is that the same amount of mass (say kg/s) must cross station 1 as crosses station 2.

Let's define "one-dimensional flow"--this is a flow in which the properties are assumed constant at every cross section (perpendicular to the flow's velocity) of the flow. By assuming 1-D flow, we neglect any variation in, for example, the velocity across a specific cross section. With these assumptions, the continuity equation reduces to the following simple result:

$$\rho_1 \, A_1 \, V_1 = \rho_2 \, A_2 \, V_2$$
$$\text{or:} \quad \rho \, AV = \text{constant}$$

Physically, the equation "says" mass is conserved. Note that the dimensions are mass per second--also, it makes sense that the mass flow varies with density, velocity, and area.

2.2.2 Incompressible and Compressible Flow

Under certain conditions, it is reasonable to assume that the flowfield is essentially "incompressible"--density remains constant. This assumption is typically made for low speeds--the following are the usual thresholds:

$$|\vec{V}| < 100 \frac{m}{s}$$

$$|\vec{V}| < 330\frac{ft}{s}$$

For a liquid (e.g. water), incompressible flow is a good assumption. Note that the above form of the continuity equation reduces to the following, if the assumption that the flow is incompressible is made:

$$AV = const$$

2.2.3 The Momentum Equation

Again, a fundamental principle is invoked to derive another extremely useful equation--this time, Newton's Second Law (F = ma) is used. Consider a differential fluid element (very, very small element of air) moving along a streamline, as shown in the following figure.
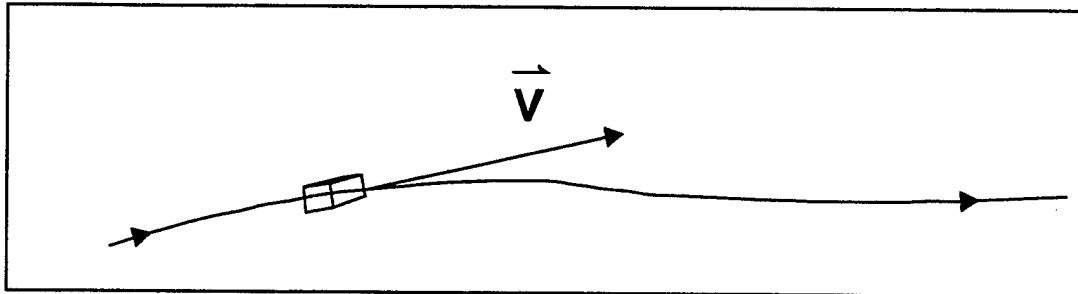
FIGURE 2.6   DIFFERENTIAL FLUID ELEMENT MOVING ALONG A STREAMLINE

Assume steady flow and neglect the weight of the fluid element--the fluid element's weight is negligible (for air) in comparison to the pressure forces "pushing" the element along the streamline. Furthermore, neglect the effects of friction between adjacent fluid particles in the flowfield. By applying Newton's Second Law to this model, F = ma reduces to the following differential equation--this equation is called "Euler's Equation":

$$dp = -\rho VdV$$

This form of the momentum equation is not convenient for solving problems relatively easily (our goal). If the flowfield is assumed to be incompressible, density can be

considered a constant, and the equation can be easily integrated between two points along a streamline, as shown below:
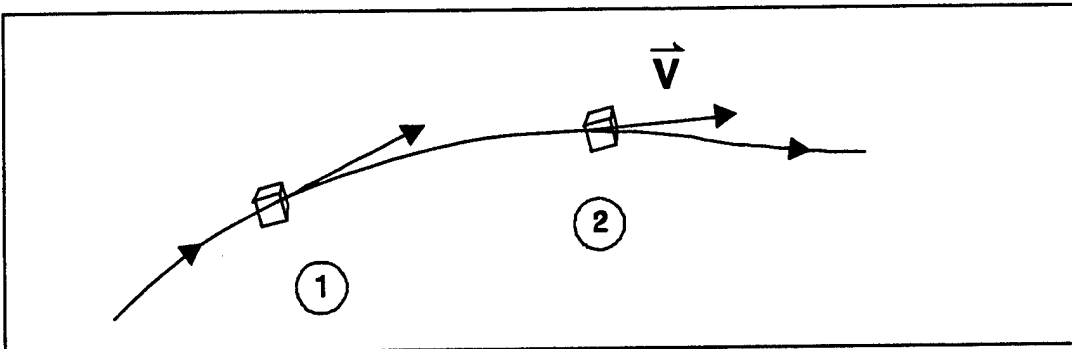
FIGURE 2.7   TWO POINTS ALONG A STREAMLINE

$$dp + \rho V dV = 0$$

$$\int_1^2 dp + \rho \int_1^2 V dV = 0$$

$$p_2 - p_1 + \rho \left[\frac{V_2^2}{2} - \frac{V_1^2}{2} \right] = 0$$

or:

$$p_1 + \frac{1}{2} \rho V_1^2 = p_2 + \frac{1}{2} \rho V_2^2$$

Let $q = \frac{1}{2} \rho V^2$ (dynamic pressure)

therefore: $p + q$ = const along a streamline (the constant is called the "total pressure")

This is called BERNOULLI'S EQUATION, one of the classics of aerodynamics. The equation is now algebraic! Remember, this did not come free--it only applies for incompressible flow. Additionally, the four assumptions of Euler's equation are still

"buried" in the result--the equation is applied along a streamline, steady flow is assumed, and forces due to weight and frictional effects are neglected.

Let's pause for a moment. Note that both the continuity and the momentum equation relate properties (pressure, density, and velocity) between, for example, points A and B in a flow. On the other hand, the equation of state can only be applied at a single point, say point A--it "says" nothing about the properties at some other location B.

2.2.4 The Speed of Sound

For a perfect gas (intermolecular forces can be neglected) the speed of sound (a) can be calculated from the following equation:

$$a = \sqrt{\gamma R T}$$

γ is the ratio of specific heats--for most aerodynamic applications, it is assumed to be a constant (1.4 for air). Note that the speed of sound for a perfect gas is only a function of temperature. The propagation of a sound wave takes place via molecular collisions--if the particles are moving faster, because they have been excited by high temperatures, then the wave speed is faster...it makes so much sense!

2.2.5 Mach Number and Aerodynamic Flight Regimes

The Mach number is defined as the ratio of the local flow velocity to the local speed of sound-it is a measure of the compressibility of the flow:

$$M = \frac{V}{a}$$

Generally, if the Mach number is less than 0.3 the flow is assumed to be incompressible. Remember, this greatly simplifies the continuity and momentum equations. Mach number is another "point property"--it can vary between points in a flow, because both velocity and temperature can, in general, vary between points in a flow. Based on the definition of the Mach number, aerodynamic flight regimes can be defined as follows:

- When the local Mach number is less than 1.0 everywhere in the flow, the flow is SUBSONIC.

- When the local Mach number is greater than 1.0 everywhere in the flow, the flow is SUPERSONIC.

- If the local velocity is equal to the local speed of sound at a point, the flow is SONIC at that point.

- When the flow has both regions of subsonic and supersonic flow, the flow is TRANSONIC. Depending upon the airplane, this occurs at freestream Mach numbers (the Mach number far ahead of the airplane) between about 0.8 and 1.2.

- With programs such as the National Aerospace Plane (NASP) hypersonic flow is currently receiving a great deal of attention. A flow is called "hypersonic" when certain physical phenomena become important that were not important at lower speeds. These include, for example, high temperature effects and relatively thin "shock layers". Typically, Mach 5 is used as the hypersonic threshold, but this value is greatly dependent upon the shape of the body of interest.

2.3 BASIC AERODYNAMICS II (Intro to Viscous Flow, Boundary Layer Concepts, Skin Friction Drag and Transition)

2.3.1 Introduction to Viscous Flow--What is it?

A viscous flow is one in which the effects of viscosity, thermal conduction, and/or mass diffusion are/is important. As a particle moves about in space, it carries with it its momentum, energy, and mass (its "identity"). Viscosity is due to the "transport" of momentum--it becomes important if there are large velocity gradients in the flow. Thermal conduction is due to the transport of energy and similarly is important in regions of strong temperature gradients. Mass diffusion is due to the transport of mass--this comes about in regions of strong concentration gradients. In subsonic aero, thermal conduction and mass diffusion are relatively insignificant--so, in this course viscous flow implies regions in which there are strong velocity gradients.

Consider a so-called "shear flow"--the streamlines are horizontal but velocity varies in the y-direction (velocity gradients) (Ref 2). Due to the transport of momentum

across the plane a-b a shear stress (τ) is exerted on plane a-b.
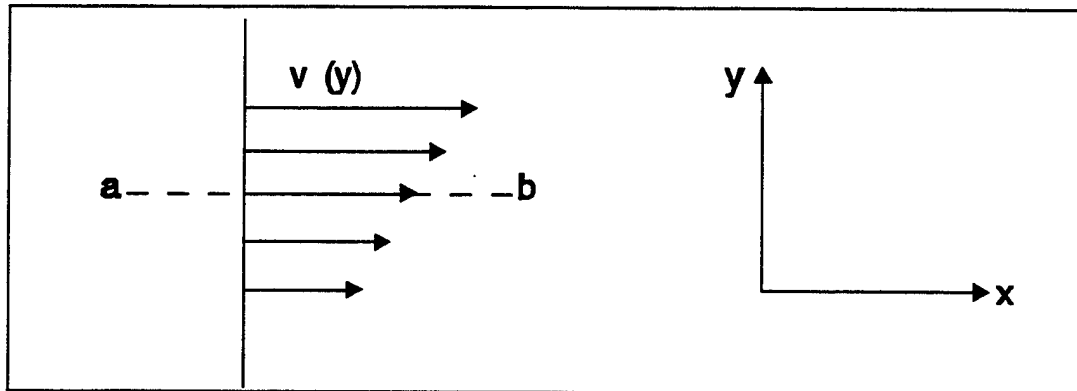
FIGURE 2.8   SHEAR FLOW

The shear stress acting on a-b is proportional to the velocity gradient as shown below: The constant of proportionality is given the symbol μ, and is called the "coefficient of viscosity" (or "dynamic viscosity", "absolute viscosity", or simply "viscosity").

$$\tau \propto \frac{dV}{dy} \quad or \quad \tau = \mu\frac{dv}{dy}$$

Physically, viscosity is a measure of a fluids resistance to shear--it has the dimensions of mass/(length)(time). Standard day sea level values are given below:

$$\mu = 1.7894 \times 10^{-5}\frac{kg}{(m)\,(s)} = 3.7373 \times 10^{-7}\frac{slug}{(ft)\,(s)}$$

For the case of air, the variation of viscosity with temperature is qualitatively sketched below. Note that as temperature increases, viscosity increases--is this what you would expect?
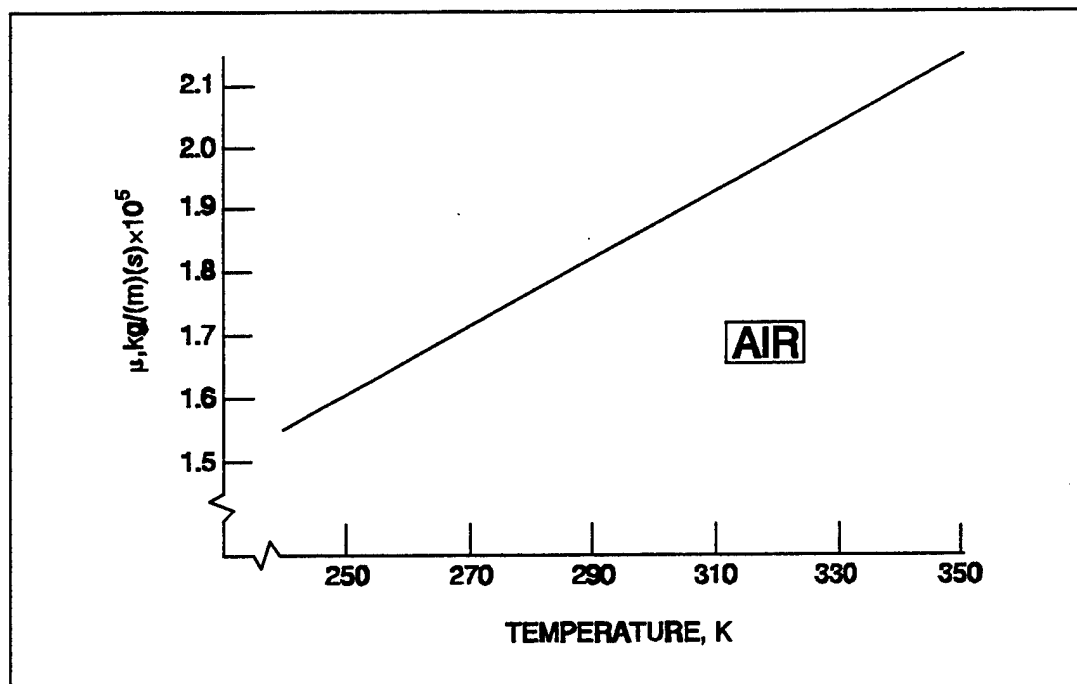
FIGURE 2.9   VARIATION OF VISCOSITY COEFFICIENT WITH TEMPERATURE (Ref. 1)

2.3.2 The Concept of a Boundary Layer

2.3.2.1 Introduction

Consider the flow over an airfoil, as shown below.
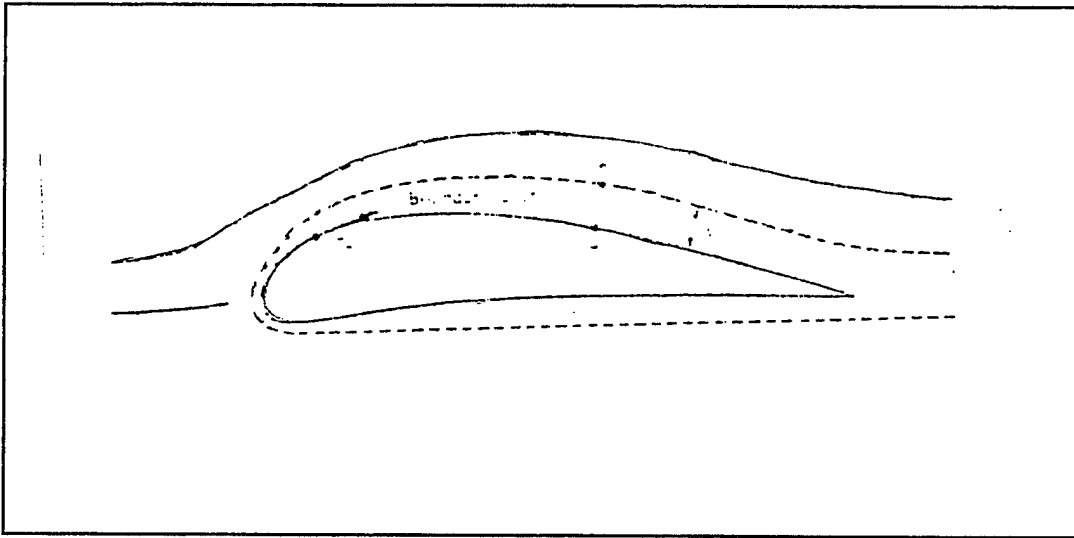
FIGURE 2.10 FLOW IN REAL LIFE, WITH FRICTION. THE THICKNESS OF THE BOUNDARY LAYER IS GREATLY OVEREMPHASIZED FOR CLARITY (Ref. 1)

There is a relatively small region adjacent to the airfoil where the effects of viscosity (friction) must be taken into account--outside this region, the flowfield is assumed to be inviscid (frictionless). This is the concept of a "boundary layer", first introduced by Prandtl in 1904. Within the boundary layer, velocity gradients are severe, and therefore viscous effects must be accounted for.

The general characteristics of a boundary layer are shown below--this is called a "boundary layer profile". Note that the velocity is zero at the surface, the so-called "no slip boundary condition".

FIGURE 2.11   BOUNDARY LAYER PROFILE

2.3.2.2 Laminar and Turbulent Boundary Layer Profiles

Laminar flow is characterized by smooth and regular streamlines--layered flow, if you will. In contrast, turbulent flow is "random, irregular, and tortuous". The two boundary layer profiles are shown below:
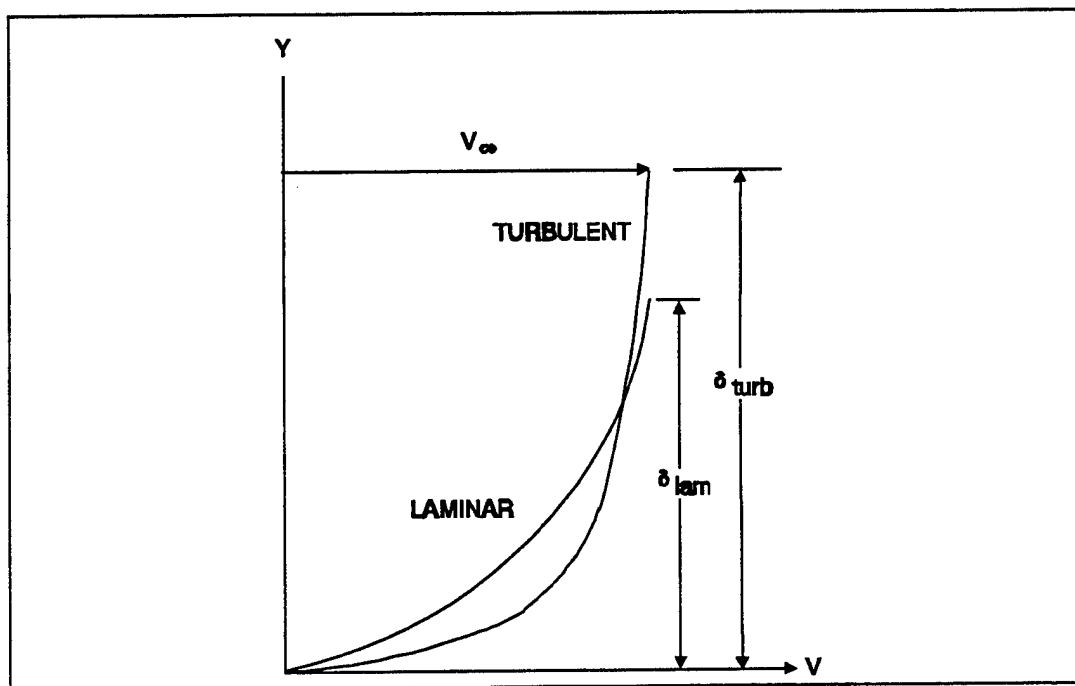
**FIGURE 2.12   VELOCITY PROFILES FOR LAMINAR AND
                TURBULENT BOUNDARY LAYERS (Ref. 1)**

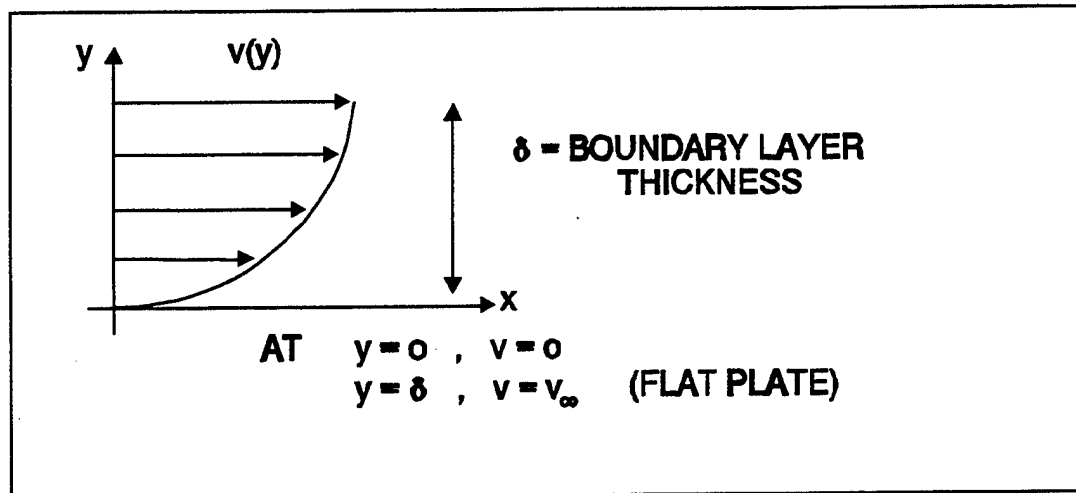Note that the velocity gradient at the wall is larger for turbulent flow. This implies that a turbulent flow causes higher shear stress and hence skin friction drag.

2.3.3 Reynolds Number

Consider the flow over a sharp flat plate as shown below--the distance x is measured from the leading edge.
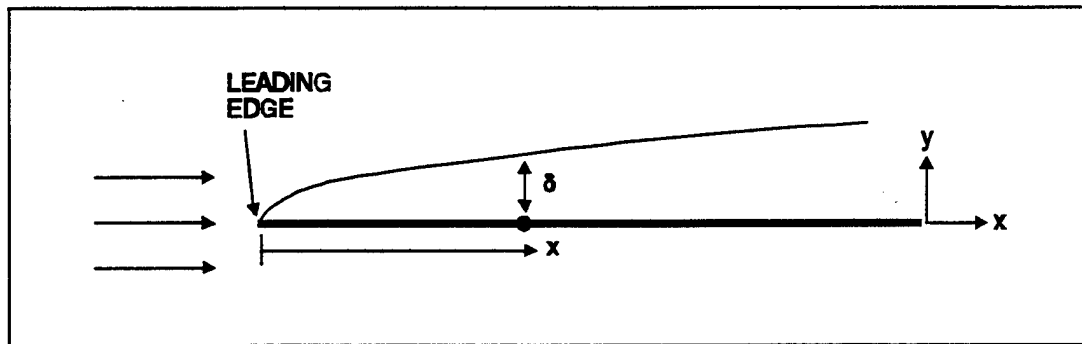
FIGURE 2.13   GROWTH OF THE BOUNDARY LAYER THICKNESS
(Ref. 1)

The Reynolds Number, based on some distance (in this case x) is defined as--the infinity subscript denotes conditions far ahead of the body:

$$Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty}$$

Physically, the Reynolds Number is a ratio of inertial forces to viscous forces--a high Reynolds Number implies inertial forces dominate viscous forces.

2.3.4 Skin Friction Coefficient and Skin Friction Drag

The definition of skin friction coefficient is given below.

$$C_f = \frac{D_f}{q_\infty S_{wet}}$$

where D_f is the "skin friction drag" and S_{wet} is the "wetted area".

Based on various experimental/theoretical results, the boundary layer thicknesses and skin friction coefficients for flow over a flat plate can be determined from the

equations below. Note that a turbulent boundary layer is thicker than a laminar--it also produces a higher skin friction coefficient, as said before.

<div align="center">

TABLE 2.1

BOUNDARY LAYER EQUATIONS

</div>

	LAMINAR	TURBULENT
δ	$\dfrac{5.2x}{\sqrt{Re_x}}$	$\dfrac{.37x}{RE_x^{1/5}}$
C_f	$\dfrac{1.328}{\sqrt{Re_L}}$	$\dfrac{0.074}{Re_L^{1/5}}$

2.3.5 Transition

Transition is defined as the "point" where the boundary layer goes from laminar to turbulent--this is sketched below:

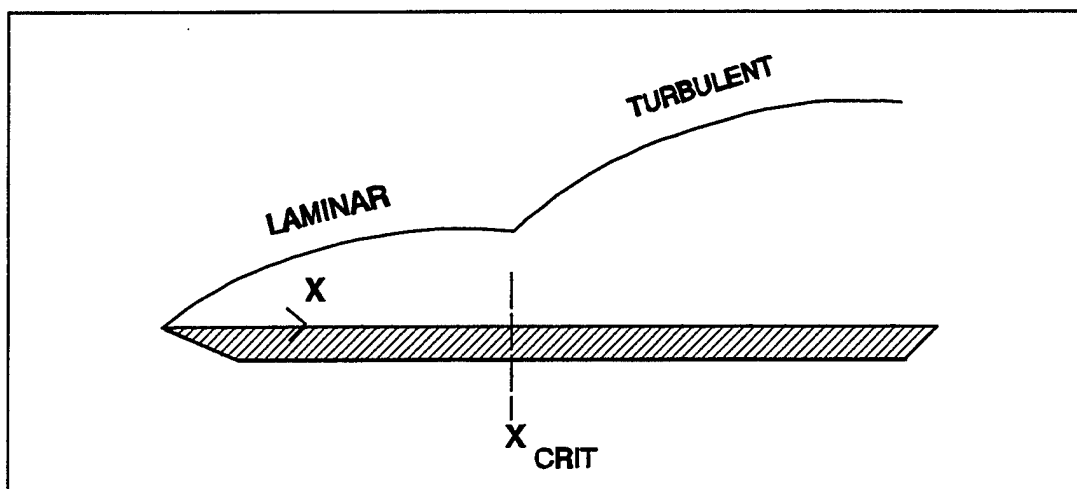**FIGURE 2.14   GROWTH OF A BOUNDARY LAYER OVER A FLAT PLATE**

The x-location where transition occurs is called "x-crit". A critical Reynolds number, based on x-crit can be defined as:

$$Re_{x_{crit}} = \frac{\rho_\infty V_\infty x_{crit}}{\mu_\infty}$$

For the case of flow over a flat plate, the critical Reynolds is approximately 500,000. We see therefore, that the critical Reynolds number provides a means to predict whether a boundary layer is laminar or turbulent at a certain x-location...very handy, but very arbitrary! Various factors can influence transition, for example surface roughness, surface temperature, Mach number, and pressure gradients (more later) in the flowfield.

2.4 BASIC AERODYNAMICS III (Pressure and Profile Drag)

2.4.1 Pressure Drag

As discussed, the presence of friction contributes to skin friction drag. Additionally, friction also causes another form of drag--this is called "drag due to flowfield separation" or simply "pressure drag". Consider the following sketch:
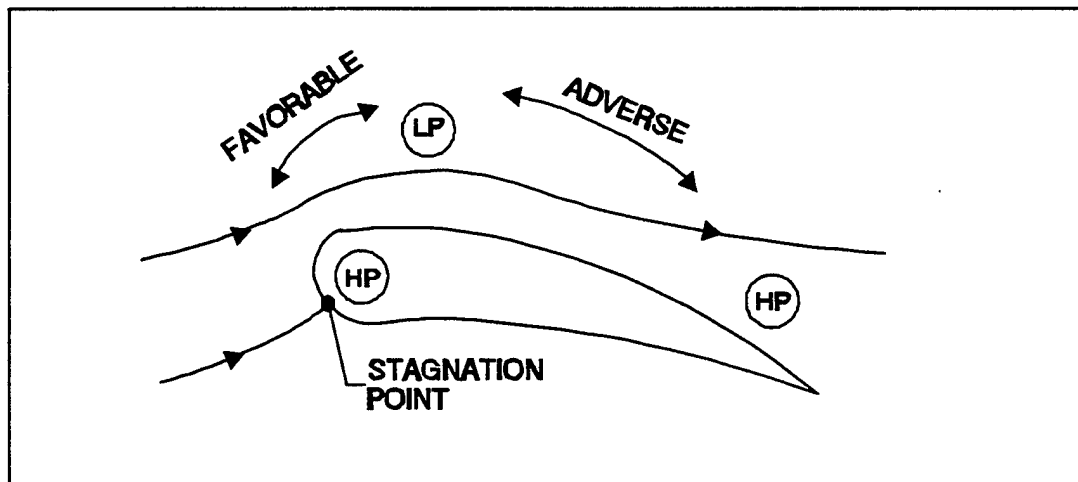

FIGURE 2.15   ILLUSTRATION OF PRESSURE GRADIENTS FOR THE FLOW OVER AN AIRFOIL

The pressure is highest at the stagnation point, the point in which the velocity stagnates (goes to zero). Over the upper surface, the velocity increases--from

Bernoulli's equation, pressure decreases to a minimum at the location of maximum velocity. Aft of the min pressure point, the velocity decreases, hence the pressure increases. When pressure increases in the streamwise direction the pressure gradient is termed "adverse". In contrast, if the pressure gradient decreases in the x-direction the gradient is called "favorable".

Why does the flow separate under certain conditions? Consider the sketch below--imagine you are "riding" on a fluid element as it goes over an airfoil.
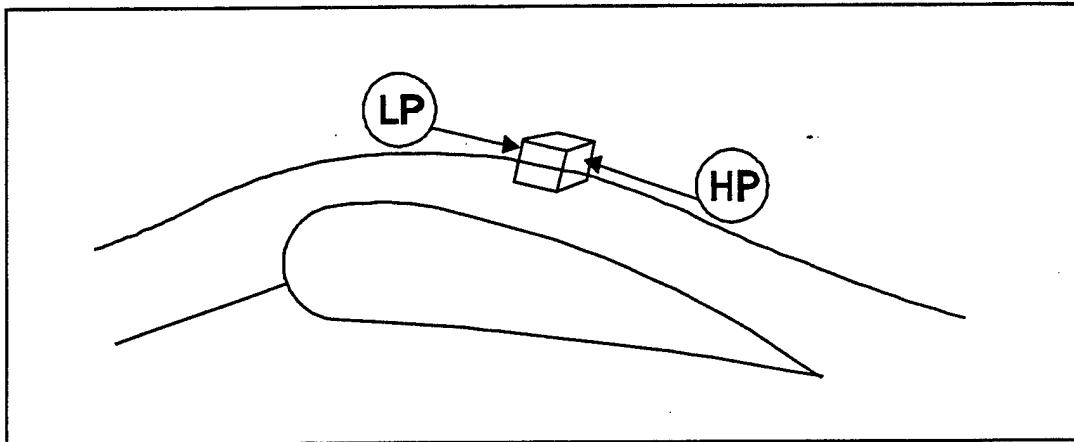
FIGURE 2.16   PRESSURE GRADIENTS AS "FELT" BY A FLUID ELEMENT

Aft of the minimum pressure point, "you" are moving into a region of relatively high pressure--the pressure gradient is adverse. Eventually, the momentum of your fluid element can no longer overcome the increasingly higher pressure...the fluid particle stops (this is called the "separation point") and ultimately reverses direction ("flow reversal"). Let's see what this looks like in terms of a series of velocity profiles.
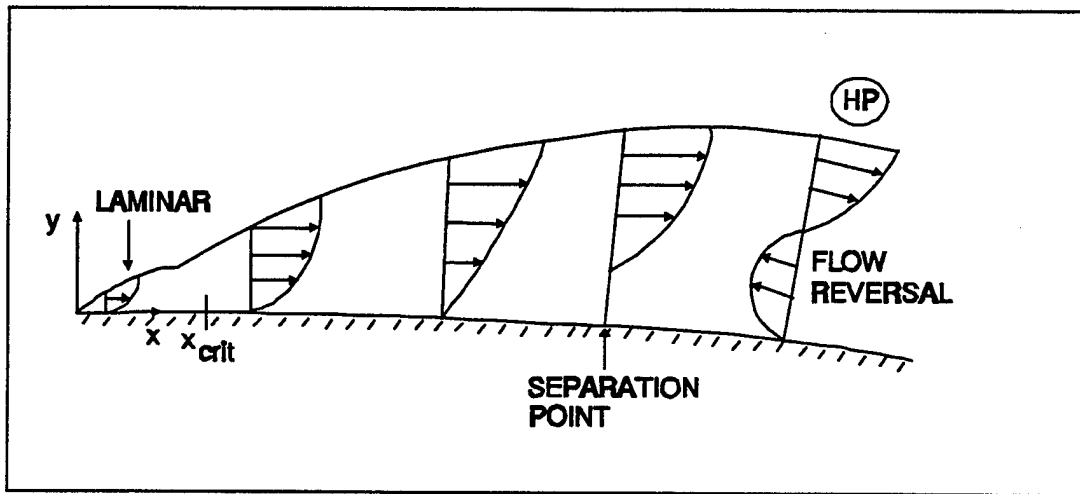
FIGURE 2.17   ILLUSTRATION OF THE PROGRESSION TOWARD FLOW REVERSAL

The consequences of flow separating over an airfoil are a drastic loss in lift (stalling) and major increase in drag. So, how is flowfield separation delayed? ANSWER: Increase the momentum, or kinetic energy, of the fluid elements within the boundary layer. This can most easily be accomplished by "tripping the boundary layer" to make it turbulent--recall that a turbulent boundary layer has a larger (than laminar) velocity gradient in the vicinity of the wall. There are several ways to "energize a boundary layer" in an effort to delay separation. Vortex generators are one example of "boundary layer control" (BLC) devices...what are some others?

2.4.2 Profile Drag

The total drag due to viscous effects is the summation of the drag due to skin friction and the drag due to flowfield separation--together they are called "profile drag".

$$D = D_f + D_p$$

or:

$$C_d = C_{d,f} + C_{d,p} = \frac{D}{q_\infty S}$$

We have arrived at one of the great compromises of aerodynamics. A laminar boundary layer decreases skin friction drag, but increases pressure drag. A turbulent boundary layer is desirable to decrease pressure drag, however, it increases skin friction drag. Ultimately, the shape of the body dictates which type of drag (skin friction or pressure) is dominant. As you would expect, skin friction is dominant for thin bodies, while pressure drag dominates blunt or "bluff" bodies. So, why does a golf ball have dimples?

As a means to check your understanding, consider the following sketch. It is a plot of drag coefficient versus Reynolds Number for the flow over a sphere.
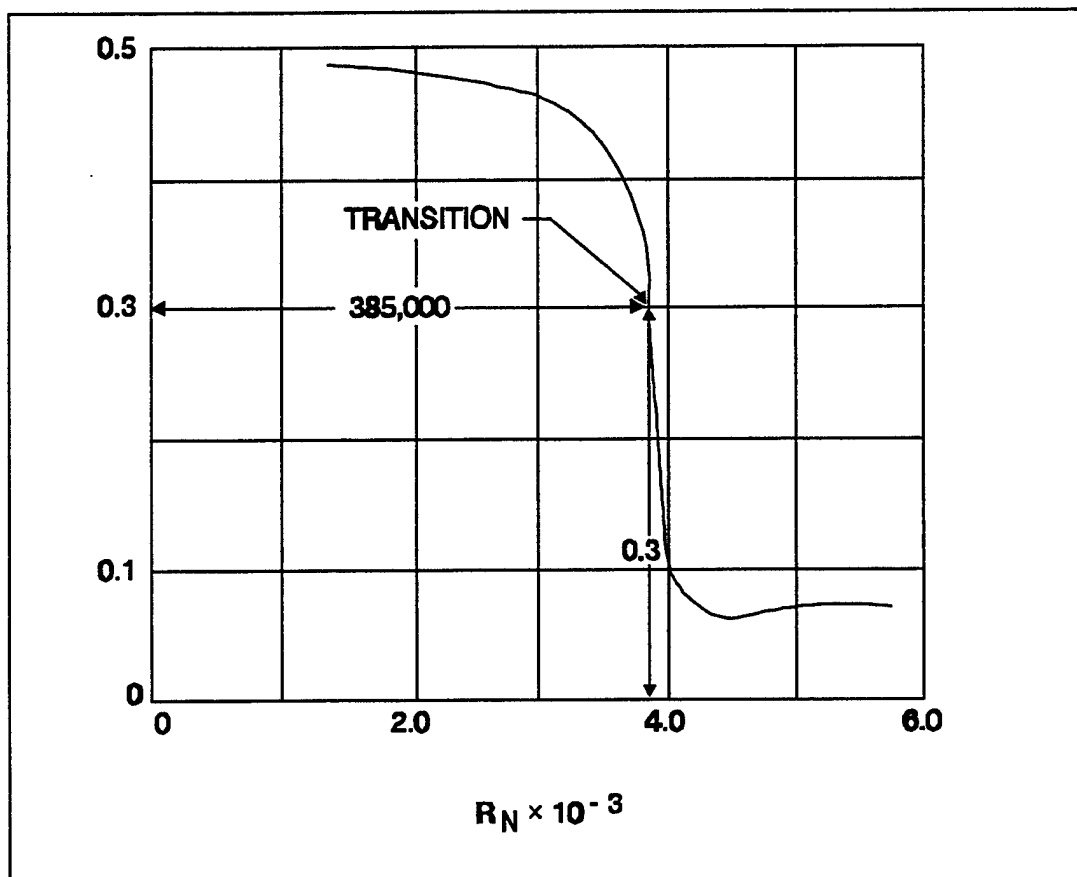
FIGURE 2.18   VARIATION OF SPHERE DRAG COEFFICIENT WITH
                REYNOLDS NUMBER (Ref. 3)
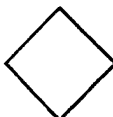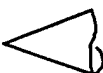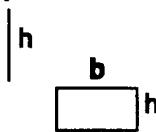
Body	Ratio		C_D based on frontal area	
Cube			1.07	
			0.81	
60% cone:			0.5	
Disk:			1.17	
Cup:			1.4	
			0.4	
Parachute (low porosity)			1.2	
Rectangular plate:	b/h	1	1.18	
		5	1.2	
		10	1.3	
		20	1.5	
		∞	2.0	
Flat-faced cylinder:	L/d	0.5	1.15	
		1	0.90	
		2	0.85	
		4	0.87	
		8	.99	

Ellipsoid:			Laminar	Turbulent
	L/d	0.75		
		1	0.5	0.2
		2	0.47	0.2
		4	0.27	0.13
		8	0.25	0.1
			0.2	0.06

TABLE 2.2
DRAG OF THREE-DIMENSIONAL BODIES AT Re $\approx 10^5$ (Ref. 4)

Note that at relatively low Reynolds Numbers the drag is on the order of 4 times the high Reynolds Numbers value. Can you explain this physically? Table 2.2 is included to get a feel for the relative magnitude of drag coefficients.

2.5 AIRFOIL TERMINOLOGY

2.5.1 Airfoil Nomenclature

Consider the following sketch of an "airfoil", or cross section of a wing. By definition, the flow over the airfoil is assumed to vary only in the x and y direction--the span is considered to approach infinity. Frequently, airfoils are called "infinite wings".
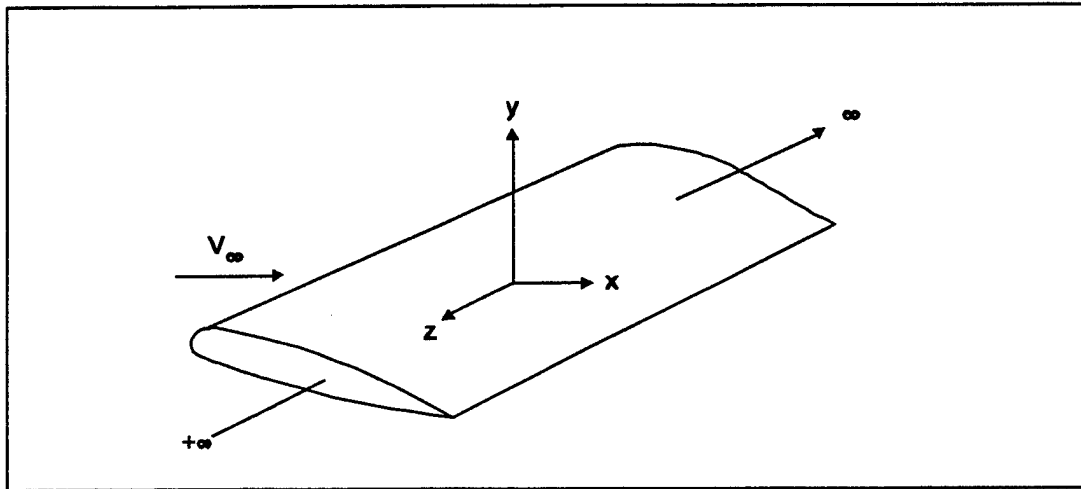
FIGURE 2.19   INFINITE (TWO-DIMENSIONAL) WING (Ref. 1)

Refer to the following figure:
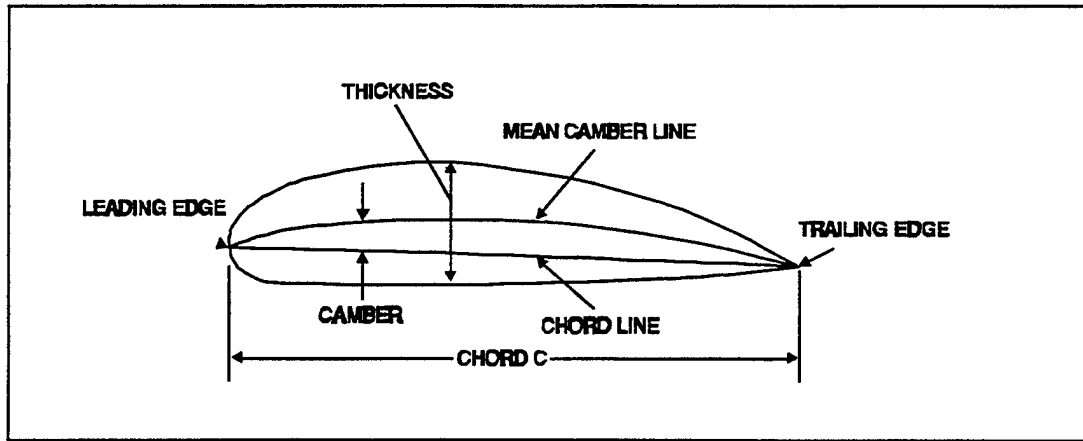
FIGURE 2.20   AIRFOIL NOMENCLATURE.   THE SHAPE SHOWN HERE IS A NACA 4415 AIRFOIL (Ref. 1)

- The straight line distance between the leading and trailing edge is called the CHORD LINE.

- The MEAN CAMBER LINE is the locus of points halfway between the upper and lower surfaces as measured perpendicular to the mean camber line itself.

- The CAMBER is the maximum distance between the mean camber line and the chord line, measured perpendicular to the chord line.

2.5.2 Center of Pressure and Aerodynamic Center

There are several ways of representing the force system on an airfoil. Typically, the forces are resolved into components parallel (drag) and perpendicular (lift) to the relative wind and a moment about the reference point of interest.

For a given angle of attack and velocity (or Reynolds Number), there is one location, called the "center of pressure" where the moments about that point are zero--refer to the following figure.
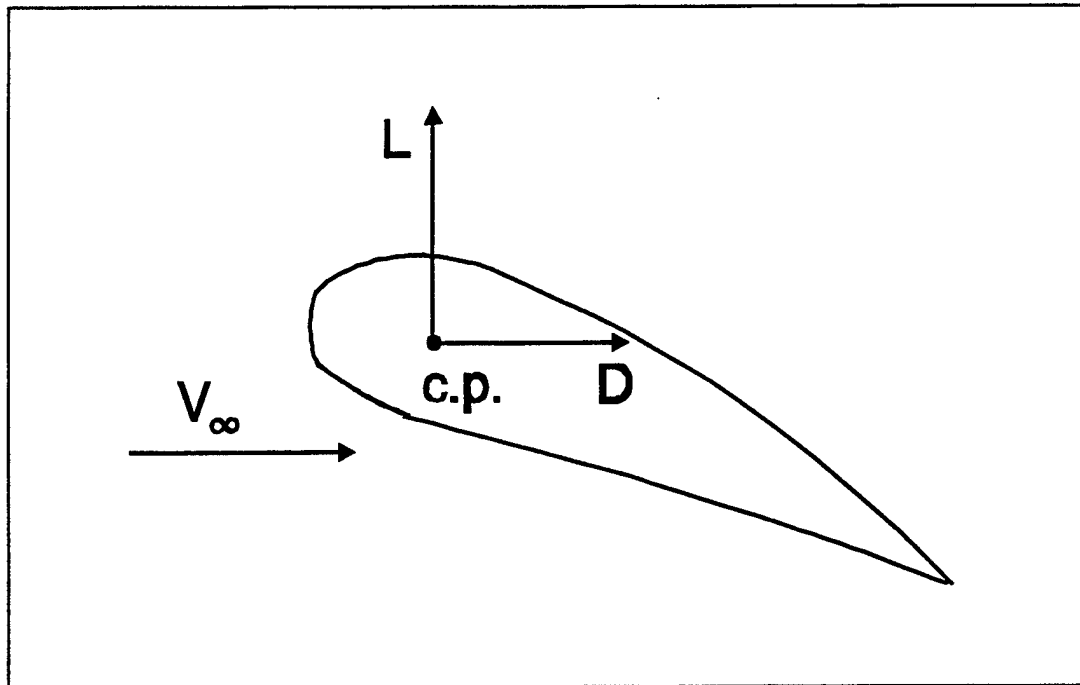
FIGURE 2.21   ILLUSTRATION OF THE CENTER OF PRESSURE

The center of pressure is not a very convenient reference point, in that a change in either angle of attack or Reynolds Number (visualize as a change in freestream velocity) will cause the center of pressure to shift on the airfoil.

There is one point on the airfoil where the moment coefficient about that point is independent of angle of attack and Reynolds Number--it is a fixed point. That reference point is called the "aerodynamic center". See the figure below. Note that pitch-up is positive.
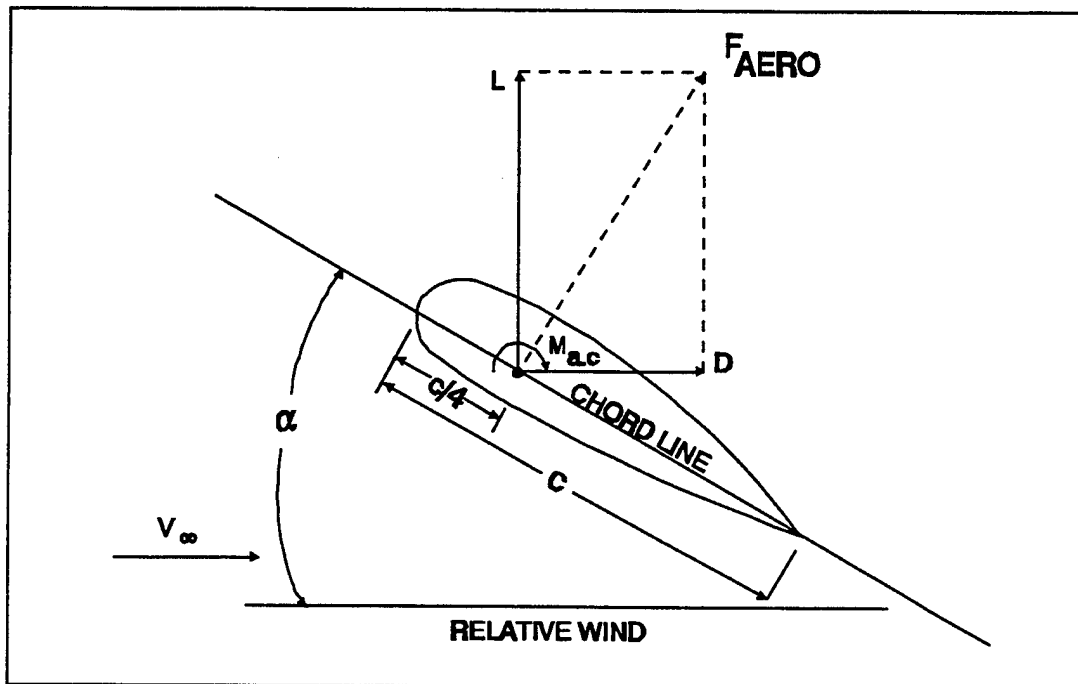
FIGURE 2.22 SKETCH SHOWING THE DEFINITIONS OF LIFT, DRAG, MOMENTS, ANGLE OF ATTACK AND RELATIVE WIND

For an airfoil in subsonic flow, the aerodynamic center is roughly at the quarter chord.

2.5.3 Lift, Drag, and Moment Coefficients

Lift, drag, and moment coefficients are frequently used to describe forces on an aerodynamic body. Their definition and significance stem from a principle called "dynamic similarity".   Consider two different flows over two different bodies. By definition, different flows are "dynamically similar" if:

-       Geometric similarity exists (scale models)
-       The "similarity parameters" are the same

If the flows are dynamically similar then:

-       The force coefficients will be equal

-       The streamline patterns will be geometrically similar

So, the key is to determine the governing similarity parameters. Dimensional Analysis (for example, the Buckingham Pi theorem) provides a mechanism to do this. By applying dimensional analysis (see the text), the following force coefficients are defined:

$$C_l = \frac{L}{q_\infty S}$$

$$C_d = \frac{D}{q_\infty S}$$

$$C_m = \frac{M}{q_\infty S c}$$

Furthermore, these coefficients are a function of three similarity parameters: Reynolds Number, Mach Number, and angle of attack. Therefore, if a scale model is placed in a wind tunnel and Reynolds Number, Mach number, and angle of attack are equal to those of a flight test, the force coefficients (lift, drag, and moment) will be the same for the two cases--a very powerful experimental tool!

2.5.4 NACA Airfoil Designations

The geometric proportions of an airfoil section are conveniently expressed in terms of three main variables:

1. Shape of the mean camber line
2. Thickness
3. Thickness distribution

A great number of airfoil sections have been developed by experimenters in the United States and elsewhere. In order to provide a reliable basis for design, the NACA, in 1929, started development of a systematic series of sections that have since provided clear proof of the influence of changes in shape of the mean line and changes in thickness of the airfoil on aerodynamic characteristics. Basing the series upon the assumption that the thickness distributions is the least important of the three previously mentioned variables, the NACA chose the average thickness distributions from two well known, successful airfoil sections as a basis for a major part of their

early tests. They were the Clark Y (USA) and the Gottengen 398 (German). By varying the percent thickness and shape of the mean camber line but keeping the thickness distribution fixed, two series of airfoil sections were developed, the original four and five digit series.

2.5.4.1 Four-Digit Series

The four-digit series is based upon a mean camber line defined by two second degree parabolas that are tangent at the point of maximum camber. The code used to define the resultant contour of the airfoil is composed of four digits: the first gives the amount of maximum camber in percent of chord, the second gives the position of the maximum camber in tenths of chord, and the last two give the maximum thickness in percent of chord. The following example may clarify the preceding explanation.

```
                        2415
                        ↑ ↑  ↑

   MAXIMUM            MAXIMUM            MAXIMUM
   CAMBER IS TWO-     CAMBER POSITION    THICKNESS IS
   HUNDREDTHS         IS AT FOUR-TENTHS  FIFTEEN-
   OF CHORD           OF CHORD           HUNDREDTHS
                                         OF CHORD
```
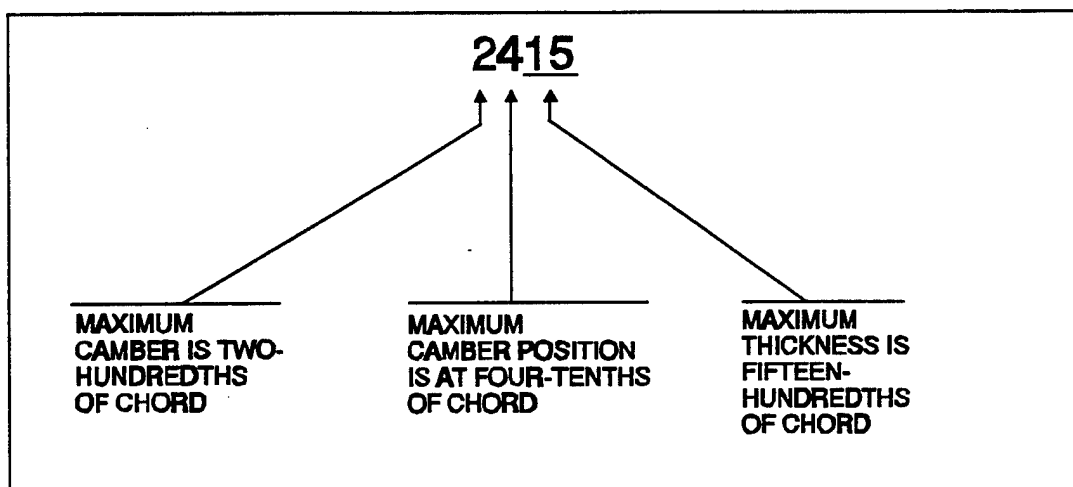
2.5.4.2 Five-Digit Series

The five-digit series has the same thickness distribution as the four digit series but is based upon a mean camber line defined by a cubic in the forward part of the airfoil, which becomes tangent either to a straight line or to an inverted cubic that forms the aft portion. The designation for the five-digit series is somewhat similar to that for the four-digit series and may be shown by the following examples:

23015

| MAXIMUM CAMBER IS TWO-HUNDREDTHS OF CHORD | MAXIMUM CAMBER POSITION IS AT THREE-TWENTIETHS OF CHORD | AFTER PORTION OF MEAN LINE IS STRAIGHT | MAXIMUM THICKNESS IS FIFTEEN-HUNDERTHS OF CHORD |

23115

| SAME AS ABOVE | AFTER PORTION OF MEAN LINE IS INVERTED CUBIC | SAME AS ABOVE |
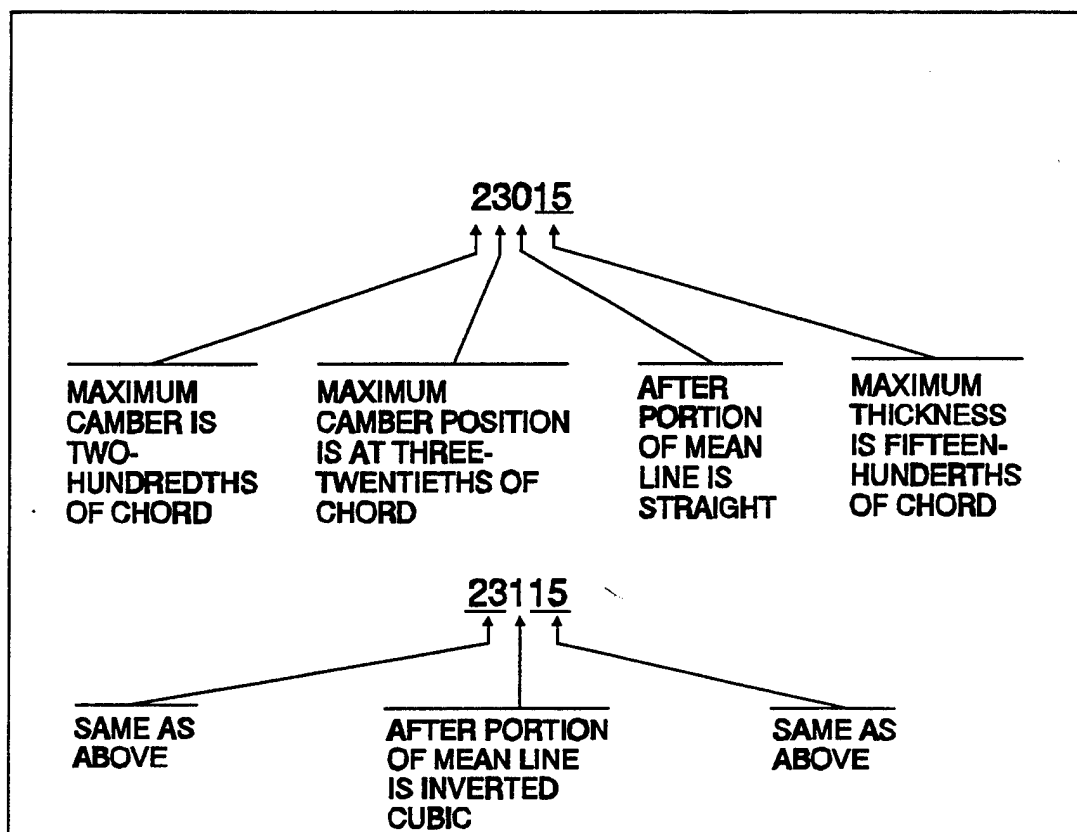
Theoretical considerations have indicated that by having the most negative pressure on an airfoil occur closer to the trailing edge than on the four-or five-digit series and by reducing the absolute value of the most negative pressure, the drag may be reduced and the high-speed characteristics improved. This led to several airfoil series that are characterized by a maximum thickness occurring somewhere in the vicinity of the mid-chord.

One of the earliest of this type was originally designed for desirable high-speed characteristics. It is a modified four-digit series in which the first four digits have their usual meaning, while a group of digits following a dash indicates thickness distribution. The first digit within this group indicates leading-edge radius according

to the following:

    0 designates sharp leading edge

    3 designates one-fourth normal four-digit radius

    6 designates normal four-digit radius

    9 designates three times normal four-digit radius

The second digit gives the position of the maximum thickness in tenths of chord. For example:

2409-04

| SAME AS FOUR-DIGIT SERIES | SHARP LEADING EDGE | MAXIMUM THICKNESS AT FOUR-TENTHS CHORD |
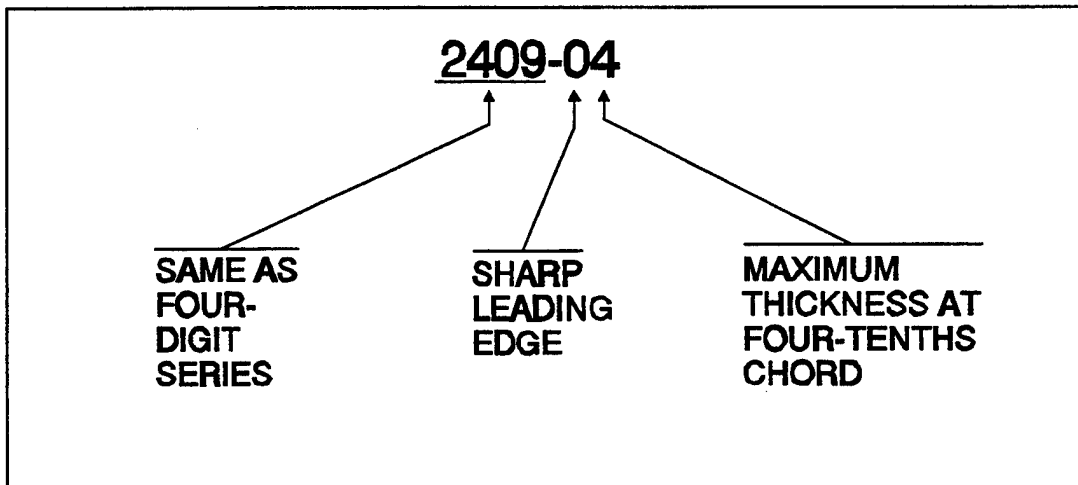
More extensive tests were subsequently run on other airfoils designed primarily for low drag characteristics, the most successful of which were the 1-series, 6 -series and 7-series. The numbering system is somewhat similar for these three groups, but each will be discussed separately.

2.5.4.3 The 1-Series Airfoils

This group is defined as a airfoil having a minimum (most negative) pressure occurring well back from the leading edge. Changes in airfoil characteristics within the series are then accomplished by changing the thickness and the shape of the mean line as in the four- and five-digit airfoils, the difference being that the airfoil is designed to give low drag at a particular angle of attack or lift coefficient. The second digit refers to the position of minimum pressure for the basic symmetrical section at zero lift, in tenths of chord. The digit after the hyphen refers tot he design lift coefficient in tenths, and the last two digits refer to the maximum thickness in percent of chord. For example:

16-218

| 1-SERIES AIRFOIL | MINIMUM PRESSURE AT SIX-TENTH OF CHORD FOR SYMMETRICAL BASIC SECTION AT ZERO LIFT | DESIGN LIFT COEFFICIENT IS 0.2 | MAXIMUM THICKNESS IS EIGHTEEN-HUNDREDTHS OF CHORD |
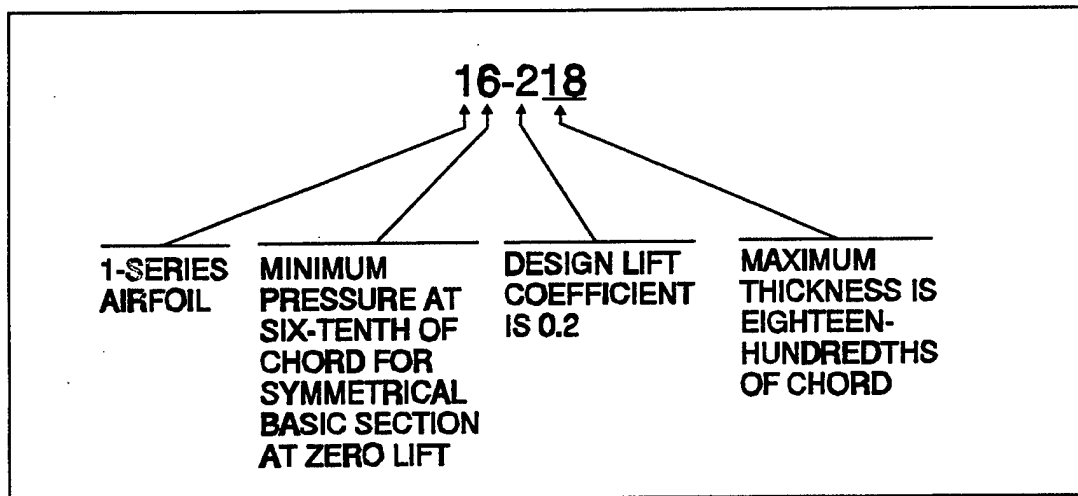
The mean camber line is usually curved in such a manner that it produces an approximately constant chordwise difference in pressure between upper and lower surfaces at the design lift coefficient, which produces an approximately constant load per inch on the chord. However, the mean line may be curved so as to produce an approximately constant load from the leading edge to the 60% chord, say, and than a linearly decreasing load to the trailing edge. This change in "loading" is indicated after the airfoil designation:

16-218, α=0.6

| SAME AS BEFORE | UNIFORM LOADING TO SIX-TENTHS OF CHORD, THEN LINEAR DECREASE TO TRAILING EDGE |
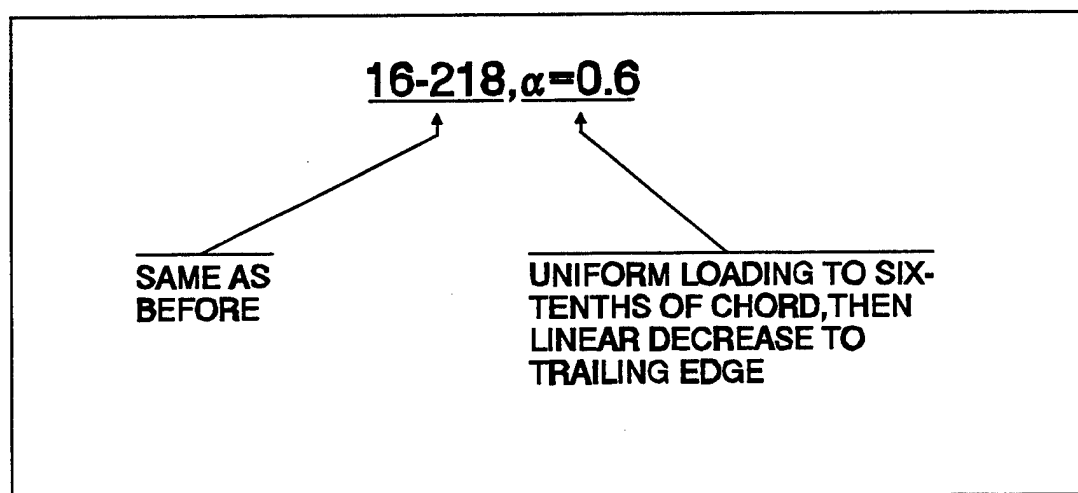
Frequently, if the loading is uniform so that a = 1.0, the load designation is deleted.

One of the most useful 1-series airfoils is the 16-( ) series, which is usually referred to as the 16-series.

2.5.4.4 The 6-Series Airfoils

The system of designation is similar to that for the 1-series airfoils, but additional information is given by another number that shows the range of lift coefficients, in tenths, above and below the design lift coefficient, for which a favorable pressure gradient exists on both surfaces.

65,3-218

| 6-SERIES AIRFIOL | SAME AS BEFORE | FAVORABLE PRESSURE GRADIENT FROM LIFT COEFFICIENT OF -0.1 TO +0.5 | SAME AS BEFORE |
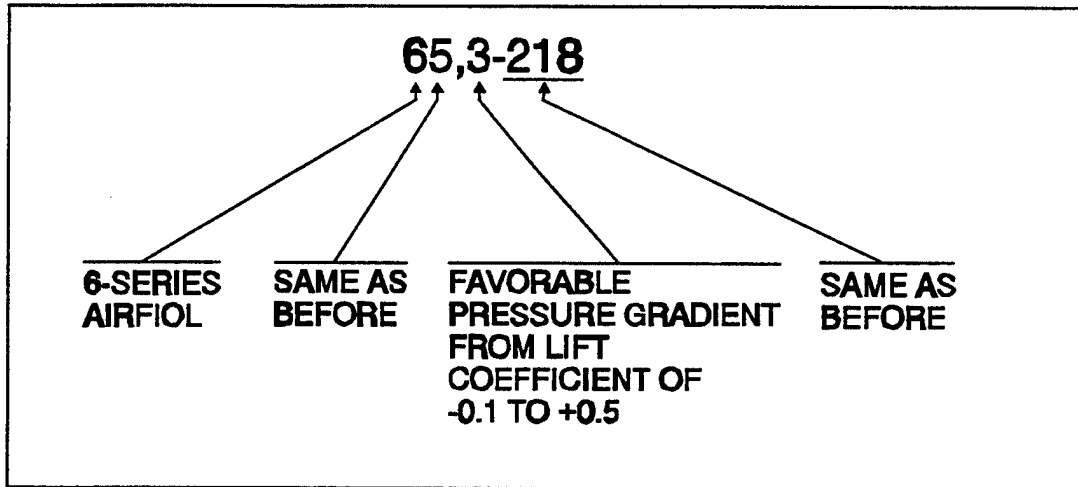
In contrast to all the preceding airfoils discussed, the thickness distribution of the above airfoil depends upon its maximum thickness. A second subgroup with a thickness distribution obtained by linearly varying all ordinates in proportion to the maximum thickness is designated as in the following example.

65(315)-218

| SAME AS BEFORE | PERCENT THICK AIRFIOL FROM WHICH THICKNESS DISTRIBUTION WAS LINELINEARLY DERIVED | SAME AS BEFORE |
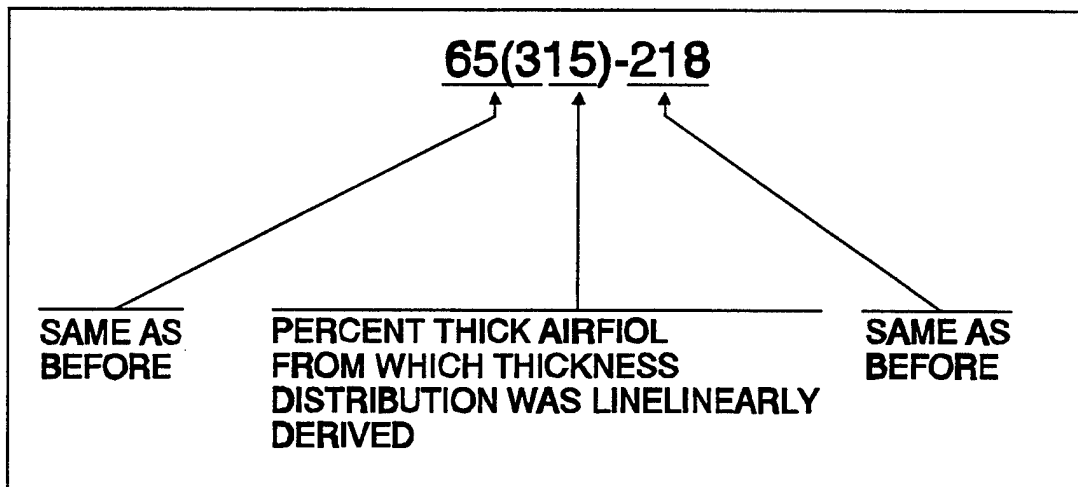
A third subgroup is based upon a theoretical rather than individually derived (first subgroup) or linearly specified (second subgroup) variation in thickness distribution with maximum thickness. All digits have the same meaning as in the first subgroup. For example:

$$65_3 \text{-}218$$

Again the above airfoil has a distribution of thickness that depends upon its maximum thickness. A fourth subgroup obtained by a linear variation like the second subgroup is specified like the second subgroup

$$65_{(315)} \text{-}218$$

All the 6-series airfoil symbols may be followed by a loading term as in the 1-series.

2.5.4.5 The 7-series Airfoils

This series was designed to produce a minimum pressure at different present chord o the upper and lower surfaces. For example:

747 A 218

| 7-SERIES AIRFOIL | FAVORABLE PRESSURE GRADIENT FOR FOUR-TENTH CHORD ON UPPER SURFACE AT DESIGN C_p | FAVORABLE PRESSURE GRADIENT FOR SEVEN-TENTHS CHORD ON LOWER SURFACE AT DESIGN C_p | SERIAL LETTER TO INDICATE THICKNESS DISTRIBU-TION AND MEAN LINE | SAME AS 6-SERIES |
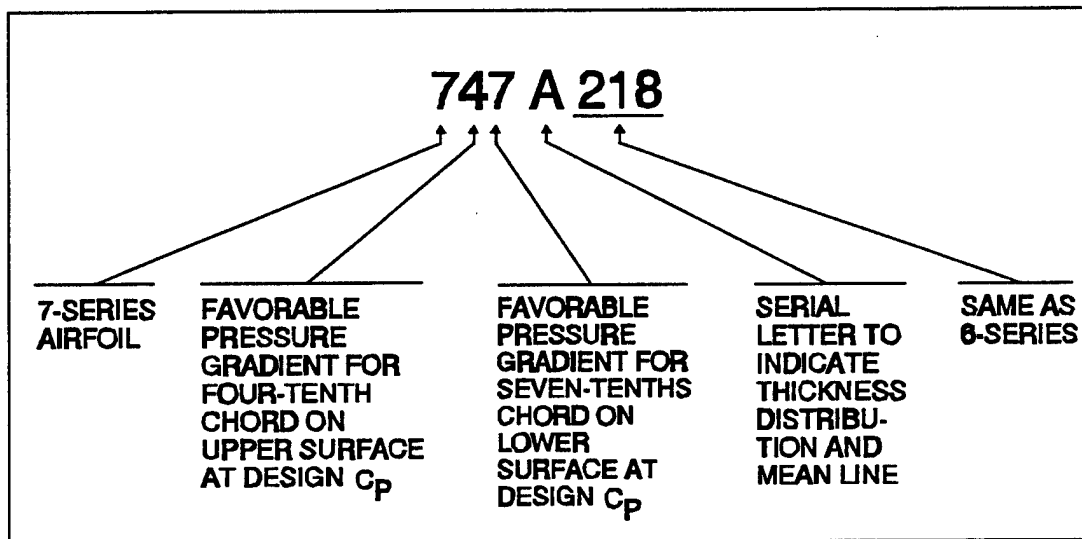
All the 7-series airfoils have thickness distributions that are individually derived, hence do not vary linearly with maximum thickness.

2.5.4.6 Supersonic Sections

Supersonic flight poses problems that are entirely different from subsonic. A series of airfoils, based on theoretical considerations, was developed by NACA. All airfoils

are characterized by a knife-edge leading edge. The numbering system may best be
illustrated by an example.

1S-(50)(03)-(50)(03)

SERIES NUMBER 1
DESIGNATES WEDGE

2 DESIGNATES
CIRCULAR-ARC

SUPERSONIC

MAXIMUM THICKNESS
OF UPPER SURFACE
IS AT 50 PERCENT
CHORD

MAXIMUM
THICKNESS OF
LOWER SURFACE
IS THREE-
HUNDREDTHS OF
CHORD

MAXIMUM
THICKNESS OF
LOWER SURFACE IS
AT 50 PERCENT
CHORD

MAXIMUM THICKNESS OF
UPPER SURFACE IS THREE-
HUNDREDTHS OF CHORD
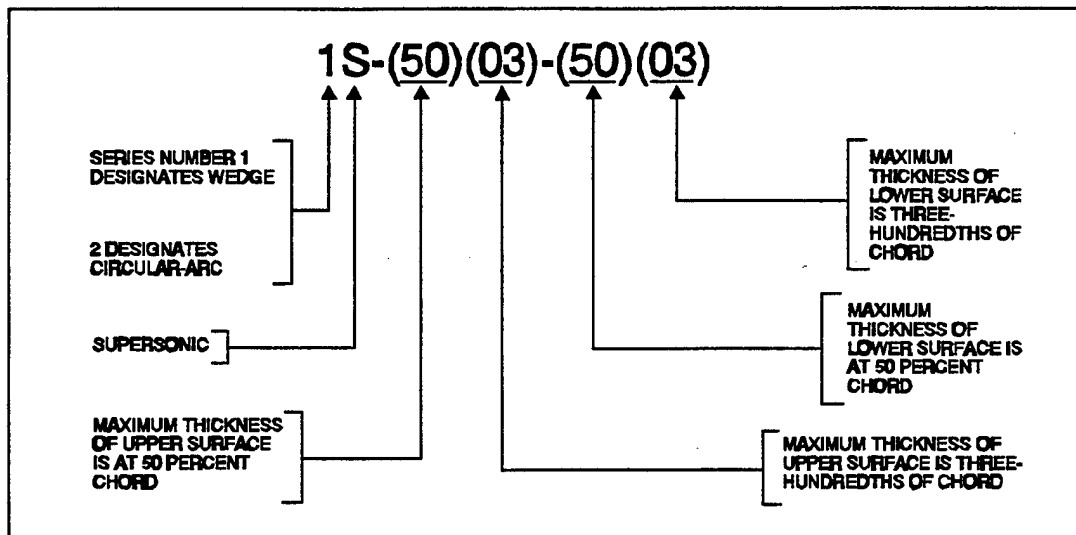
Examples of some common airfoils of the various groups described are depicted in
Figure 3.3 (3.1:61-67).
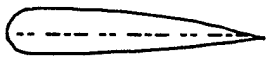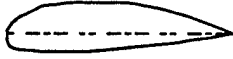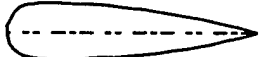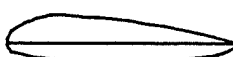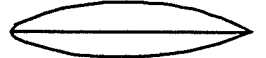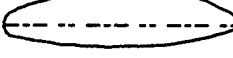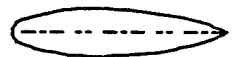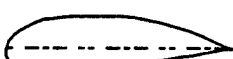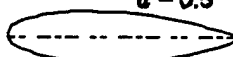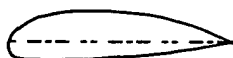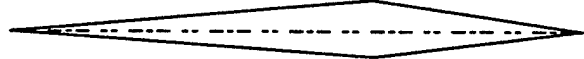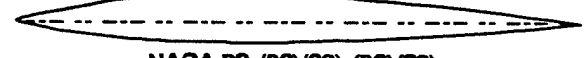
FOUR-DIGIT SERIES AIRFOILS	NACA 0015	NACA 4415
FIVE-DIGIT SERIES AIRFOILS	NACA 23015	NACA 23115
1-SERIES AIRFOILS	NACA 16-015	NACA 16-515
6-SERIES AIRFOILS	NACA 65, 2-015 / NACA 65$_2$-415	NACA 65(216)-415 $\alpha = 0.5$ / NACA 65$_{(421)}$-415
7-SERIES AIRFOILS	NACA 747A015	NACA 474A415
SUPERSONIC AIRFOILS	NACA 1S-(70)(03)-(70)(03) / NACA 2S-(30)(03)-(30)(03)	

FIGURE 2.23   EXAMPLES OF COMMON AIRFOIL SECTIONS

2.5.4.7 Other Airfoil Designation Systems

This extensive coverage of the NACA airfoil designation system should not be construed to mean that it is the only system in existence. There are several other systems in existence. Examples are the Gottingen (German), the St. Cyr (French) and

other types of systems and special airfoils developed in other countries and even by specific aircraft manufacturers.

The NACA system has been in existence for over 50 years, and many NACA airfoils are still used throughout the international aerospace industry.

2.5.5 Airfoil Data

Representative data and key points about airfoil data are presented in this section. Recall, we are dealing with airfoils ("infinite wings")--to avoid confusion, lower case subscripts are typically used to denote airfoil data. Later, upper case subscripts will be used to denote finite wing data/airplane data.

2.5.5.1 Lift Coefficient versus Angle of Attack

The sketch below is typical of a lift coefficient versus angle of attack plot for an airfoil section--refer to Appendix D in the text.
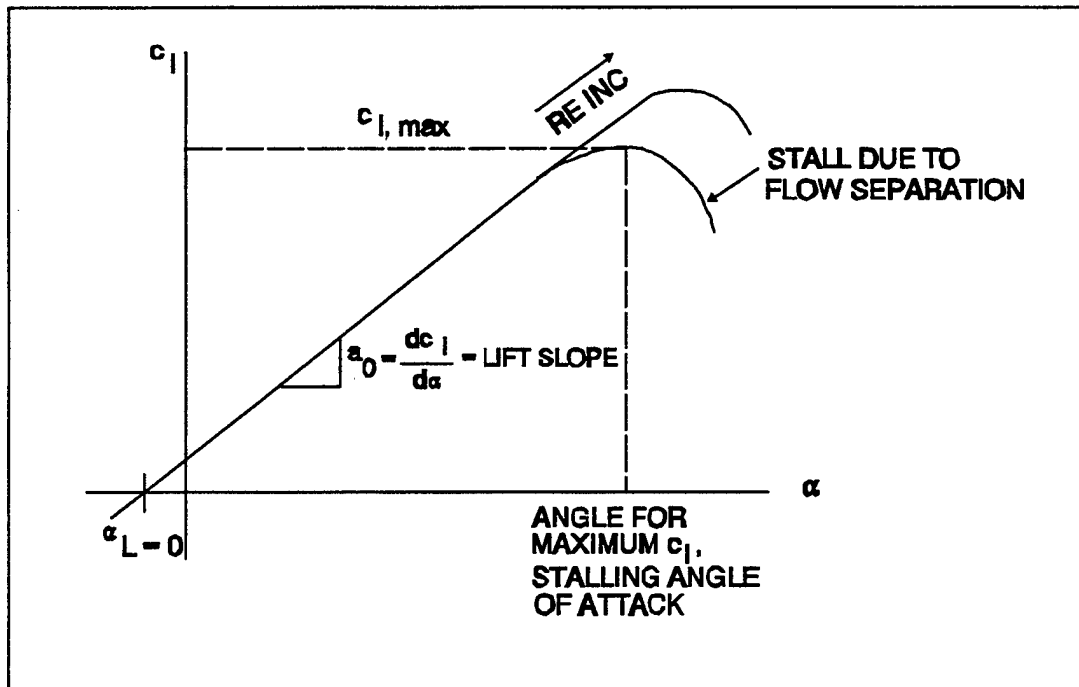
FIGURE 2.24   SKETCH OF A TYPICAL LIFT CURVE

A couple key notes about these figures:

    1. Refer (above) to the angle of attack at zero lift (denoted by $\alpha_{L=0}$)--for the airfoil to generate zero lift, this is the required angle of attack. For a positively cambered airfoil this is a negative angle of attack. Note that at zero degrees angle of attack, lift is generated--that's usually desirable!

2. The "lift curve slope" (a_o) is roughly 0.1/degree or 2π per radian--you aero majors will recognize these numbers as classic thin airfoil theory results.

3. $C_{l, max}$ is the maximum lift coefficient obtainable before the airfoil stalls--the corresponding angle of attack is called "alpha stall" (angle of attack in which the airfoil stalls).

4. Remember, lift coefficient is a function of angle of attack, Reynolds Number, and Mach Number. This plot gives the variation with angle of attack and Reynolds Number--later Mach number will be addressed. Note, that an increase in Reynolds Number (all else constant) tends to increase max lift--based on our discussion, does this make sense?

If the airfoil is symmetric (not cambered, as in the above case) the curve shifts--what do you think it looks like?

2.5.5.2 Drag Coefficient versus Lift Coefficient

The figure below is typical of a so-called airfoil "drag polar"--lift coefficient versus drag coefficient. Because lift coefficient and angle of attack vary linearly (before stall), visualize lift coefficient as angle of attack.
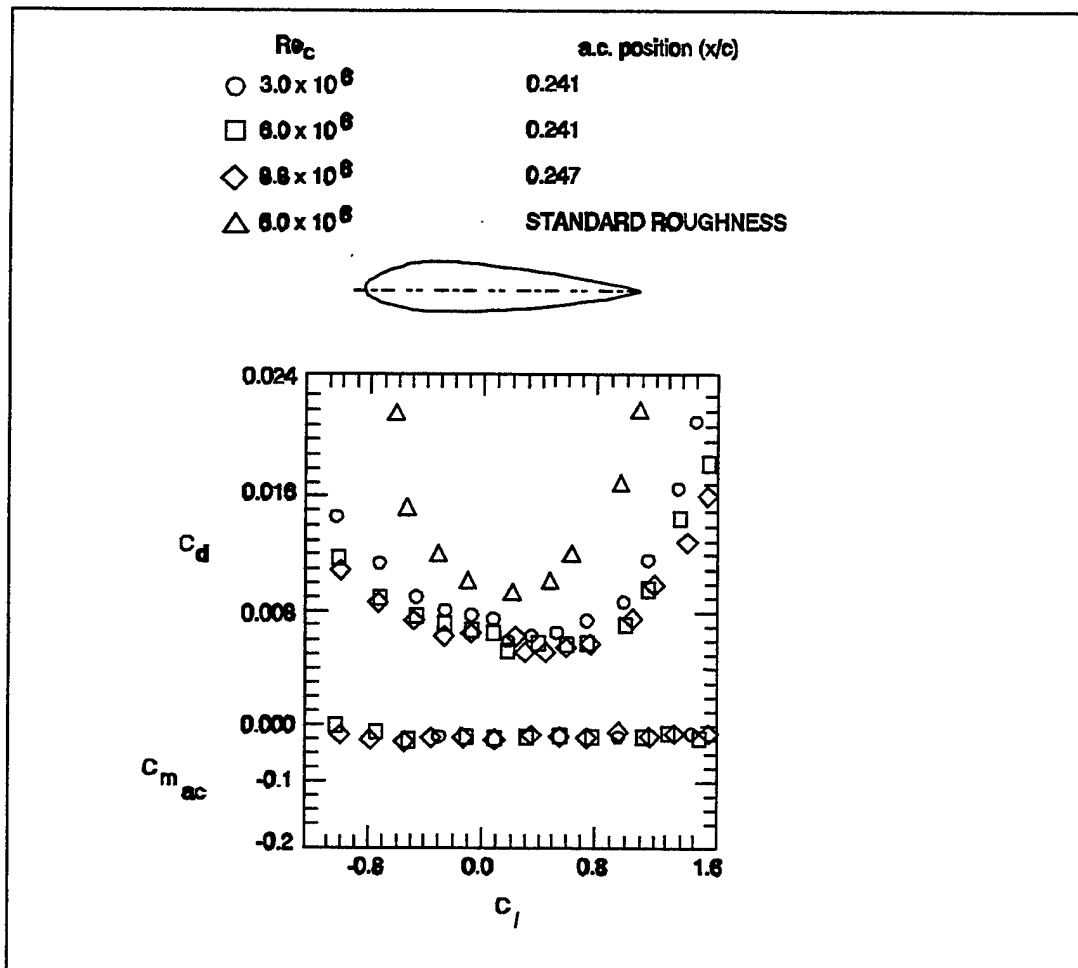
FIGURE 2.25   SECTION DRAG COEFFICIENT AND SECTION MOMENT COEFFICIENT (WITH RESPECT TO THE AC) FOR AN NACA 23012 AIRFOIL.   (DATA FROM REF 6)

Again, a few key points:

    1.   For a symmetric airfoil, the drag polar is symmetric about the "y" axis---does this make sense?

    2.   At low angles of attack, skin friction drag is dominated. The flowfield "sees" a relatively thin body. However, at high alphas (high lift coefficients), pressure drag

dominates--the flowfield "sees" a blunt body, and drag "blows up" as the flow begins to separate.

3. What is the effect of Reynolds Number on drag coefficient. Does this make sense?

2.5.5.3 Moment Coefficient about the Aerodynamic/Quarter Chord Versus Lift Coefficient.

Refer again to the above figure. Key points are:

1. The moment coefficient about the a.c. is insensitive to changes in Reynolds Number and angle of attack--this is exactly what we said earlier.

2. The values of the coefficients are negative--this implies that the airfoil is tending to pitch downward. In stability control, this has important ramifications.

3. Looking at the data in Appendix D, it's clear the a.c. is roughly located at the quarter chord location.

2.5.6 Pressure Coefficient

Another convenient dimensionless ratio is the "pressure coefficient". It is defined below:

$$C_p = \frac{P - P_\infty}{q_\infty} = \frac{P - P_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$$

For cases in which Bernoulli's equation is valid (low speed) the pressure coefficient can be rewritten as follows:

$$P_\infty + \frac{1}{2}\rho V_\infty^2 = p + \frac{1}{2}\rho V^2$$

$$P - P_\infty = \frac{1}{2}\rho (V_\infty^2 - V^2)$$

or:

$$C_p = \frac{\frac{1}{2}\rho (V_\infty^2 - V^2)}{\frac{1}{2}\rho_\infty V_\infty^2} = 1 - \left(\frac{V}{V_\infty}\right)^2$$

The above is a very convenient result--it shows the following:

      1. At the stagnation point, where the velocity is zero, the pressure coefficient is +1.0.

      2. On the upper surface, where generally the velocity is higher than freestream (lower pressure), the pressure coefficient is negative.

      3. On the lower surface, where generally the velocity is lower than freestream (higher pressure), the pressure coefficient is positive.

      4. The pressure coefficient is zero when the local velocity is equal to the freestream velocity.

The following is a typical pressure coefficient plot versus distance (nondimensional) along the chord. As shown, the area under the curve represents the lift coefficient (at that specific angle of attack).
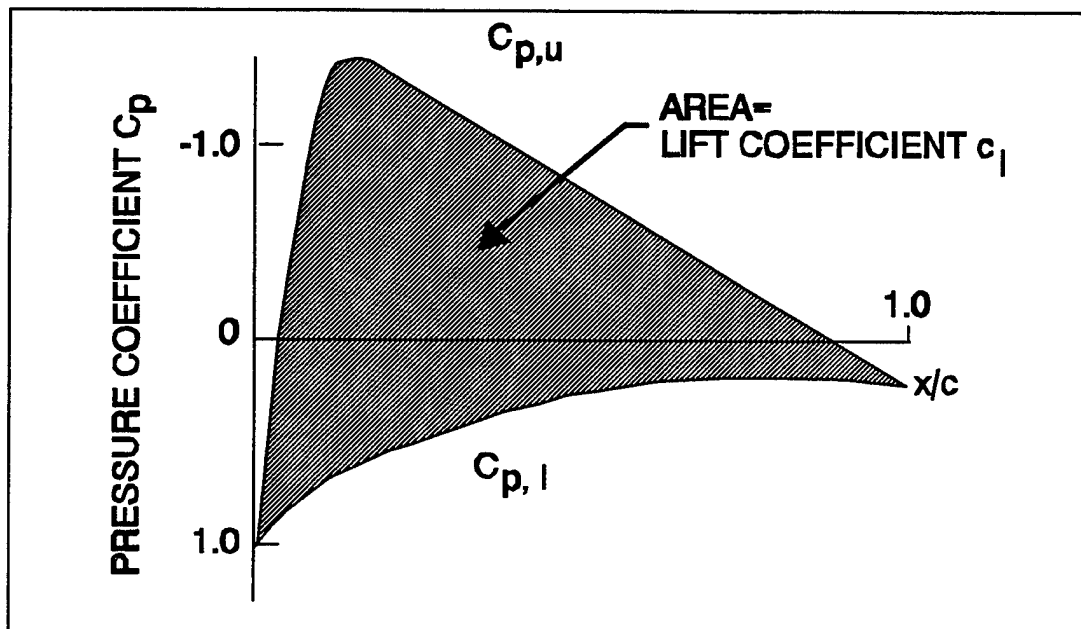
FIGURE 2.26   SKETCH OF THE PRESSURE COEFFICIENT OVER THE UPPER AND LOWER SURFACES OF AN AIRFOIL SHOWING THAT THE AREA BETWEEN THE TWO CURVES IS THE LIFT COEFFICIENT

2.5.7 Laminar Airfoils

Since maintaining laminar flow reduces skin friction drag, much effort has been expended in the past and much is going on today to achieve this goal. Laminar flow airfoils are created using one or more of several design techniques.

The first design criterion is that the airfoil be smooth, because rough spots or bumps trigger turbulence and consequently create turbulent flow in the boundary layer. Current efforts are being made to develop "super smooth" surfaces, mainly through the use of composite materials.

Another important factor is the location of the maximum thickness to chord ratio, t/c, of the airfoil with respect to the distance from the leading edge. Locating the maximum thickness further aft, at approximately 40 to 60% chord moves the minimum pressure point farther from the leading edge and creates a long, shallow, negative (favorable) pressure gradient as shown below.
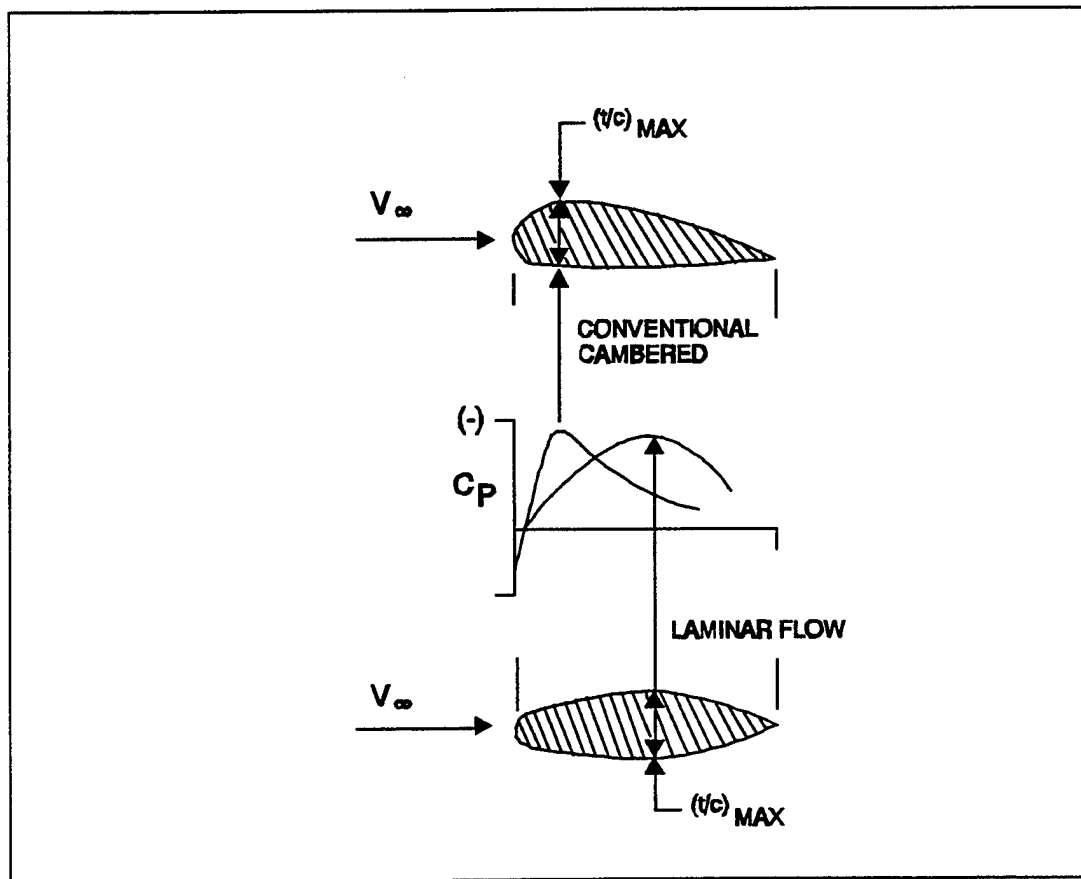
FIGURE 2.27   COMPARISON OF CONVENTIONAL CAMBERED AND LAMINAR WING SECTIONS

This favorable pressure gradient tends to delay boundary layer transition to turbulent flow. The maximum thickness point cannot be located too far aft, or an increase in the total drag due to separation will negate the decrease in skin friction drag. The reason separation is more likely to occur is because of the increased steepness of the adverse pressure gradient aft of the maximum thickness point as shown in Figure 2.27.

Designing an airfoil with a small thickness to chord ratio reduces the local velocity in the boundary layer, which in turn reduces the local Reynolds number. This causes the critical Reynolds number to occur father aft of the leading edge of the airfoil, which promotes the desired goal of keeping the flow in the boundary layer laminar father along the surface; however, the airfoil cannot be too thin. It must be thick enough to provide a useable negative pressure gradient which can develop sufficient lift and is large enough to help delay transition.

Another design feature to be considered for a useable laminar airfoil is the shape of the leading edge. Experiments have shown that a parabolic shape is more desirable than sharp or circular leading edges. Because of the relative thinness of laminar flow airfoils, care has to be taken with leading edge radius design to prevent leading edge stall at high angles of attack. Leading edge stall is generally considered disastrous.

A laminar flow airfoil exhibits a decrease in skin friction drag through a rather restricted range of lift coefficients. Usually the airfoil is designed to give the greatest decrease in skin friction drag at low C_L's, or at high speeds. The drag polar for a laminar flow airfoil with the typical "drag bucket" is compared to the drag polar for a nonlaminar airfoil in Figure 2.28.

By varying the camber of the airfoil, the designer can place the drag bucket around the desired lift coefficient without sacrificing any of the effects of laminarization.
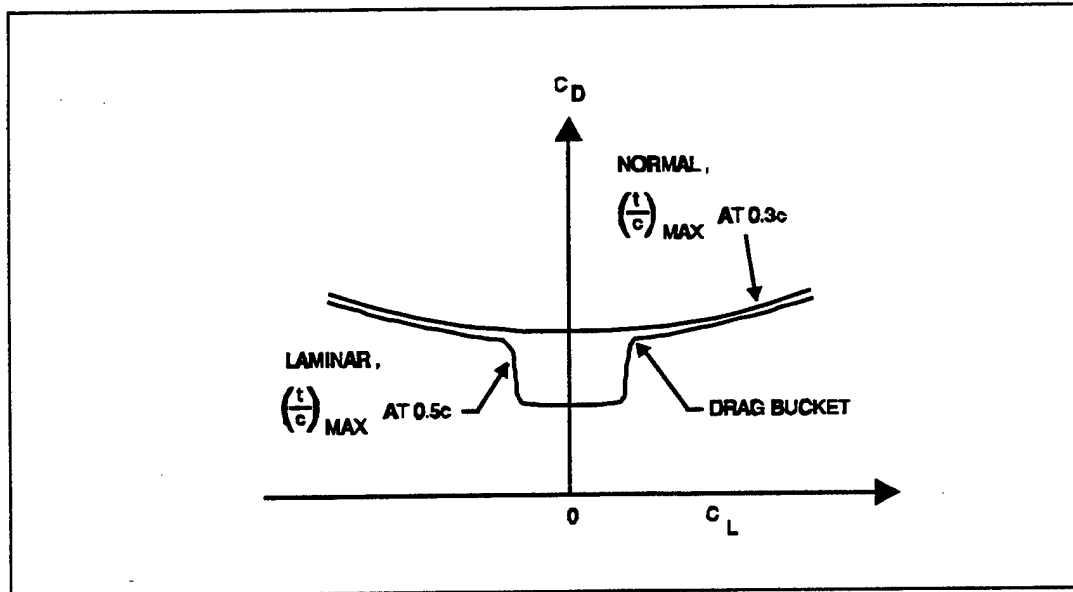
FIGURE 2.28   LAMINAR FLOW WING SECTION DRAG POLAR

Unfortunately, in spite of very encouraging wind tunnel results which yield laminar flow drag polars, significant drag reduction in-flight has not been realized. Laminar flow cannot be maintained for any significant distance along either a fuselage or a wing surface, and the drag bucket does not appear on drag polars derived from flight test data. Laminar flow airfoils are historically interesting and are widely discussed in the academic literature, but to date results of their use have been disappointing from a practical point of view.

However, if laminar flow could be maintained on just an aircraft wing, the reduction in skin friction drag would be sufficient to greatly increase aircraft range. The idea of using suction to keep a laminar boundary layer attached and stable was tried on the X-21 by Northrop about 1960. Spanwise slots were milled into a jet transport wing, and suction was applied. Although the concept was proven sound, the airplane suffered from the practical difficulties of keeping the highly unstable laminar boundary layer from transitioning to a turbulent layer or separating from the wing. The scheme was finally abandoned, but the concept promises great gains in long range cruise performance if it can be successfully employed. Today, laminar flow control is a high cost, high risk, but potentially high payoff gamble.

Unfortunately, most design methods used to reduce skin friction drag tend to cause early boundary layer separation. This creates pressure drag which is often much larger than the reduction in skin friction drag.

2.6 COMPRESSIBILITY EFFECTS

2.6.1 Compressibility Correction for Lift Coefficient--The Prandtl-Glauert Rule

Pause for a moment...we said earlier that lift coefficient is a function of angle of attack, Reynolds Number, and Mach Number. We have discussed the influence of the first two. What happens to the lift coefficient as Mach Number increases? The following compressibility correction is used to modify low speed data (for example in Appendix D) in order to account for the effects of compressibility--it is called the "Prandtl-Glauert Rule":

$$C_l = \frac{C_{l,o}}{\sqrt{1-M_\infty^2}}$$

$C_{l,o}$ is the incompressible value (Appendix D) of the lift coefficient. The Prandtl-Glauert Rule is a reasonable correction until the airfoil reaches the "critical Mach number" (discussed in the next section).

2.6.2 Critical Mach Number and Critical Pressure Coefficient

As freestream Mach number is increased over an airfoil, the velocity over the upper surface increases--likewise does the Mach number. At some freestream Mach number the flow over the upper surface first reaches sonic condition--the corresponding freestream Mach number is called the "critical Mach number". The pressure coefficient at the point of sonic flow is called the "critical pressure coefficient".

2.6.3 Lift Coefficient Variation with Mach Number

The following is a qualitative sketch showing the variation of lift coefficient with Mach number.
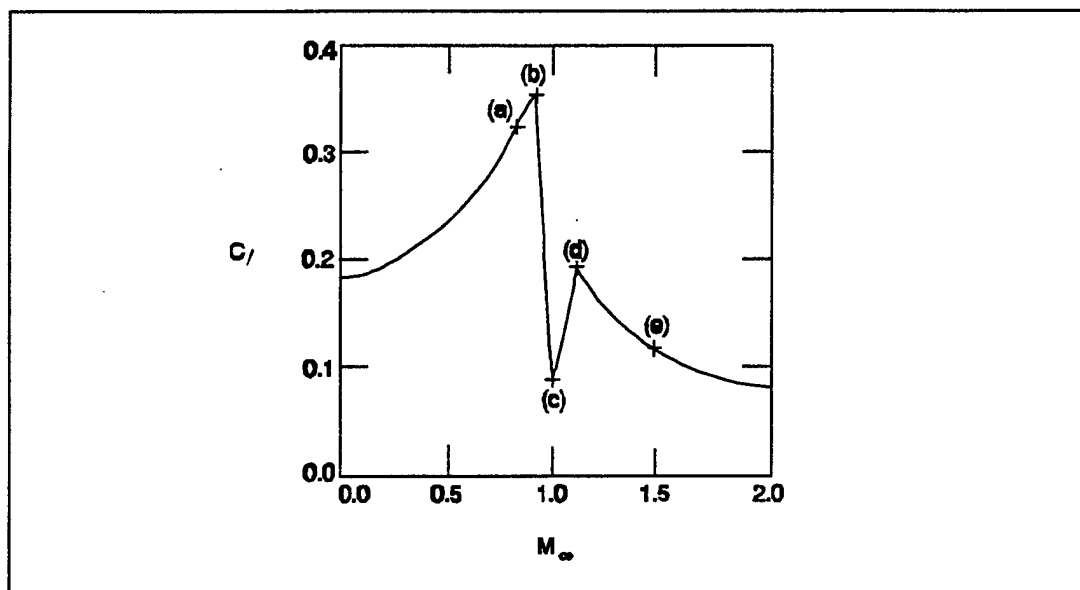
FIGURE 2.29   LIFT COEFFICIENT AS A FUNCTION OF MACH
NUMBER   TO   ILLUSTRATE   THE   EFFECT   OF   COMPRESSIBILITY.
(FROM REF. 6)   REFER   TO FIG 2.30   FOR THE FLOW FIELDS
CORRESPONDING TO THE LETTERED POINT ON THIS GRAPH

To understand the above trends, refer to the following figure--it shows the variation
in the flowfield about the airfoil as the freestream Mach number is increased.

Flow field around an airfoil in transonic streams based on schlieren photographs:
(a) Mach number $M_x = 0.75$; (b) Mach number $M_x = 0.81$; (c) Mach number $M_x = 0.89$;
(d) Mach number $M_x = 0.98$; (e) Mach number $M_x = 1.4$. (From Ref. 9.3.)
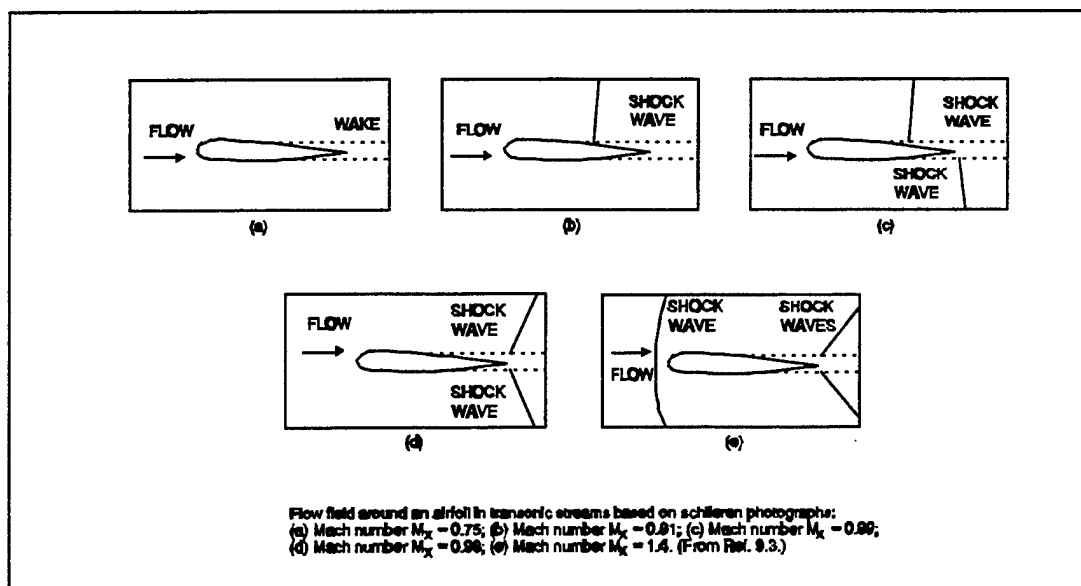
FIGURE 2.30   FLOW FIELD AROUND AN AIRFOIL IN TRANSONIC
STEAMS BASED ON SCHLIEREN PHOTOGRAPHS:

The key points are as follows:

1. At point a, the flow is subsonic. Prandtl-Glauert provides a reasonable estimate of the lift coefficient.

2. At b, the flow is supersonic over most of the upper surface, terminating in a shock wave--pressure increases across a shock, thus causing increased likelihood of the flow separating from the airfoil's surface (adverse pressure gradient is created).

3. At c, the flow over the lower surface is essentially all supersonic, implying a lower pressure than for case b (flow on the upper surface is relatively unchanged). The net effect is a lower (than at b) lift coefficient.

4. Between c and d the shock waves, on both surfaces, move aft--the lift coefficient increases.

5. Further increase in Mach number results in a "bow wave"--velocity decreases across this initial shock resulting in relatively weaker shocks at the trailing edge. Lift coefficient decreases due to the change in the velocity/pressure distribution over the airfoil.

2.6.4 Drag Coefficient Variation with Mach Number

The following sketches qualitatively show how/why drag coefficient varies with Mach number:
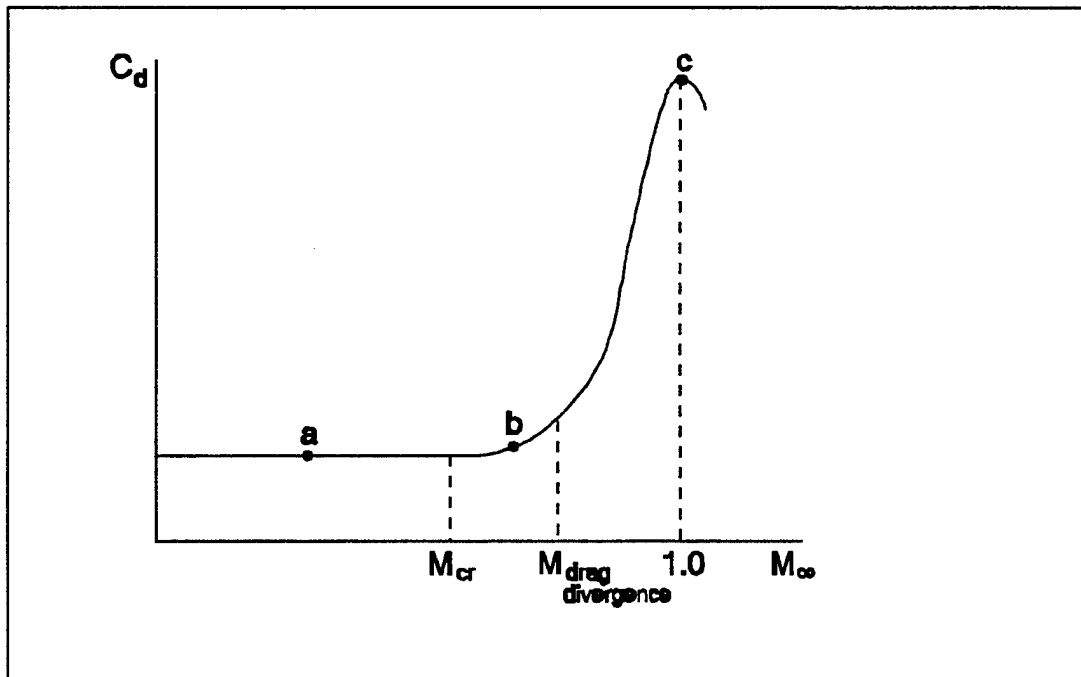
FIGURE 2.31 VARIATION OF DRAG COEFFICIENT WITH MACH NUMBER (Ref. 1)
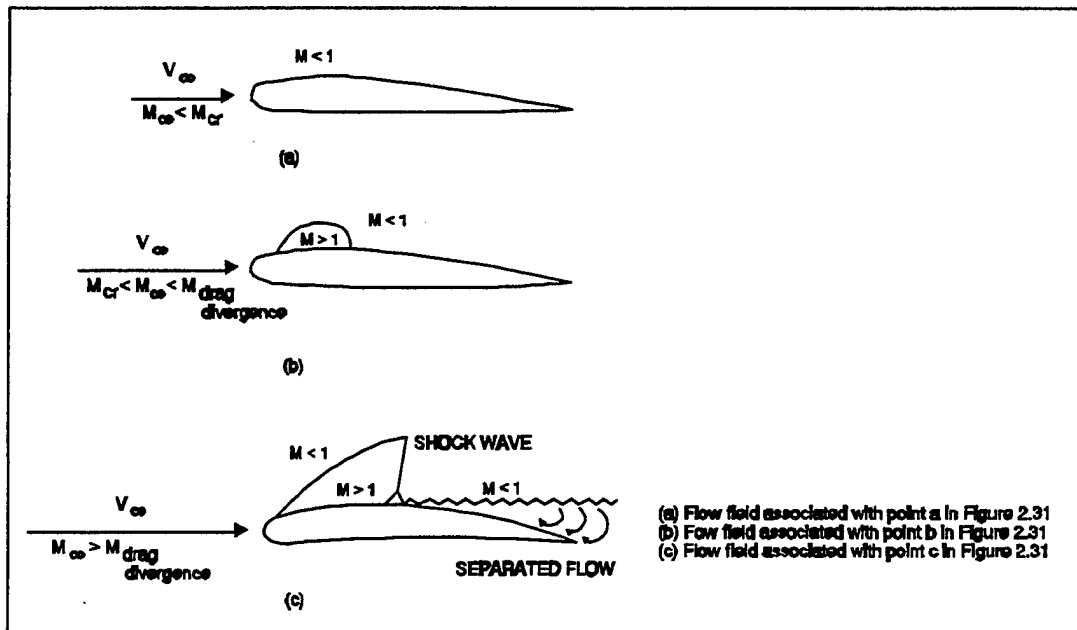
FIGURE 2.32 PHYSICAL MECHANISM OF DRAG DIVERGENCE
(Ref. 1)

At some freestream Mach number, the drag coefficient increases rapidly--that freestream Mach number is called the "drag divergence Mach number".

2.6.5 Wave Drag

At supersonic speeds, an airfoil (or other body) "drags" with it shock waves. Because static pressure increases across these waves, the pressure distribution over the body is altered. The net effect is a "new" form of drag, called "wave drag".

2.6.6 Summary of Airfoil Drag

Compressibility considerations have introduced a "new" form of drag, wave drag. Therefore, the total drag on the airfoil now consists of three contributors, as shown below:

$$C_d = C_{d,f} + C_{d,p} + C_{d,w}$$

2.7 FINITE WINGS

2.7.1 Infinite Versus Finite Wings

To this point, we have only considered ourselves with airfoils, or "infinite wings". What are the effects of wing tips? First, some definitions dealing with finite wing geometry.

2.7.1.1 Wing Terminology

The following terms are used to describe the geometric and aerodynamic characteristics of wings.

Wing Area (S) is the reference wing area used in computing aerodynamic coefficients. In the US, it is generally the main "trapezium" area as shown shaded in Figure 2.33.
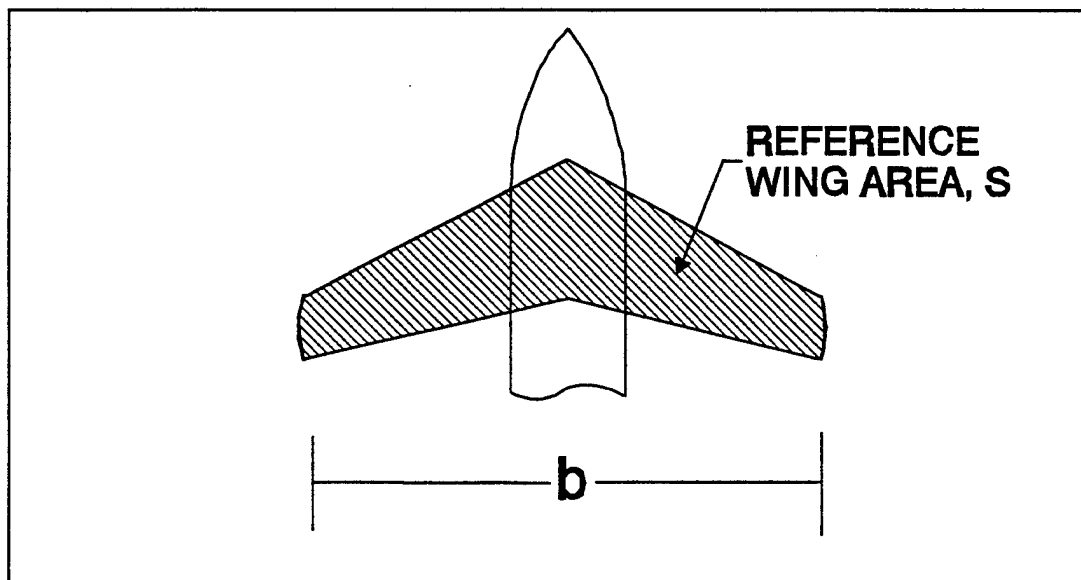
FIGURE 2.33   REFERENCE WING AREA

Wing Span (b) is also a reference dimension measured wingtip to wingtip as shown in Figure 2.33.

Mean Aerodynamic Chord (MAC) is a theoretical chord for an imaginary, straight, untapered wing which has the same force vectors as the actual wing throughout the speed range of concern. The true mean aerodynamic chord is difficult to compute for many modern aircraft wings since they can be tapered, swept, elliptical or otherwise irregular.

In practice, it has been found that the MAC can be approximated geometrically as shown in Figure 2.34.

Taper Ratio (λ) is the ratio of the tip chord to the root chord of a wing.

Sweep Angle (Λ) is the angle the quarter chord line of the wing makes with a lateral line perpendicular to the fuselage reference line. Sweep aft is normally considered positive, and forward sweep is negative (Refer to Fig 2.35).

Aspect Ratio (AR) for any wing planform is defined as the ratio of the span squared to the area (b^2/S).
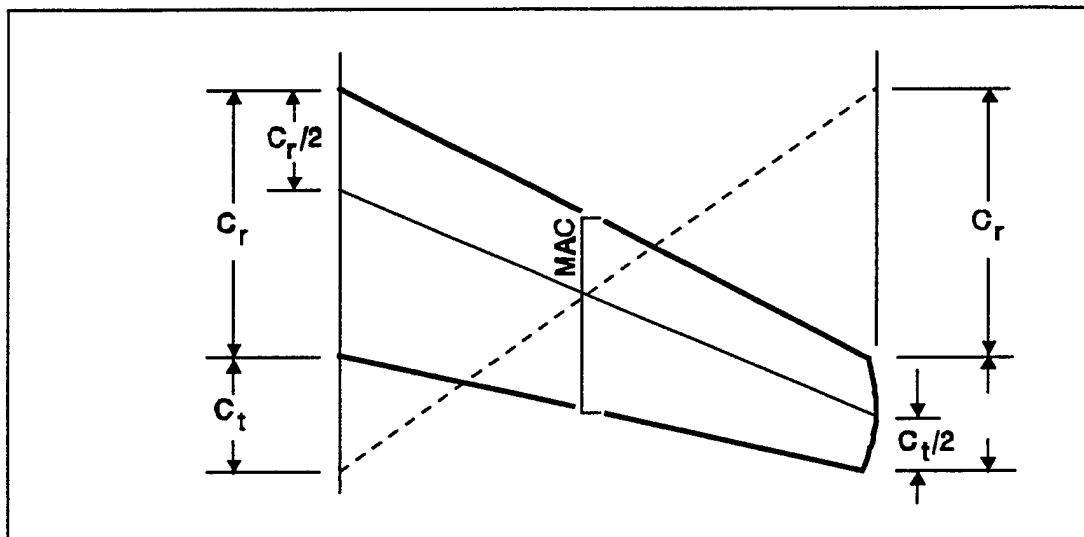
FIGURE 2.34   MEAN AERODYNAMIC CHORD DETERMINATION

Figure 2.35 summarizes geometric characteristics.

S = WING AREA, SQ FT
b = SPAN, FT
c = CHORD, FT

AR = ASPECT RATIO
AR = b/c
AR = b^2/s

C$_r$ = ROOT CHORT, FT
C$_t$ = TIP CHORT, FT
λ = TAPER RATIO
λ = C$_t$/C$_r$

Λ = SWEEP ANGLE, DEGREES
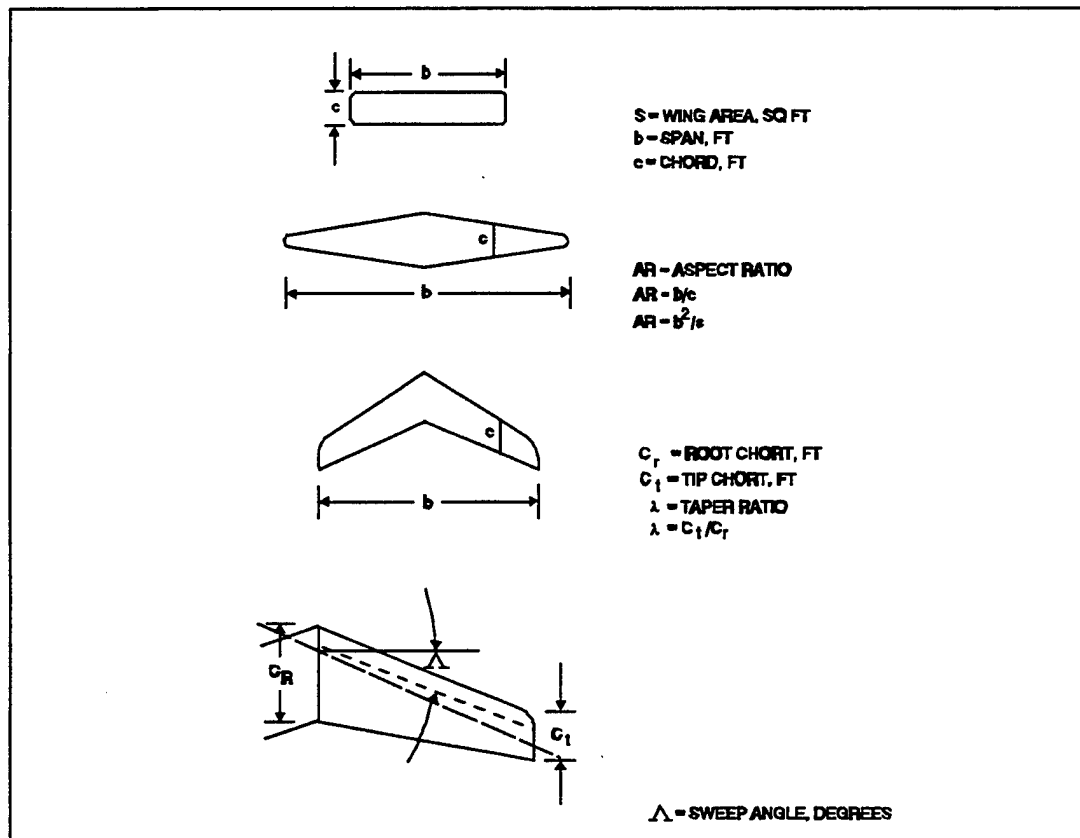
**FIGURE 2.35   SUMMARY OF WING GEOMETRIC CHARACTERISTICS
(Ref. 7)**

2.7.2 The Origin of Downwash

Consider the following sketch of a finite wing:
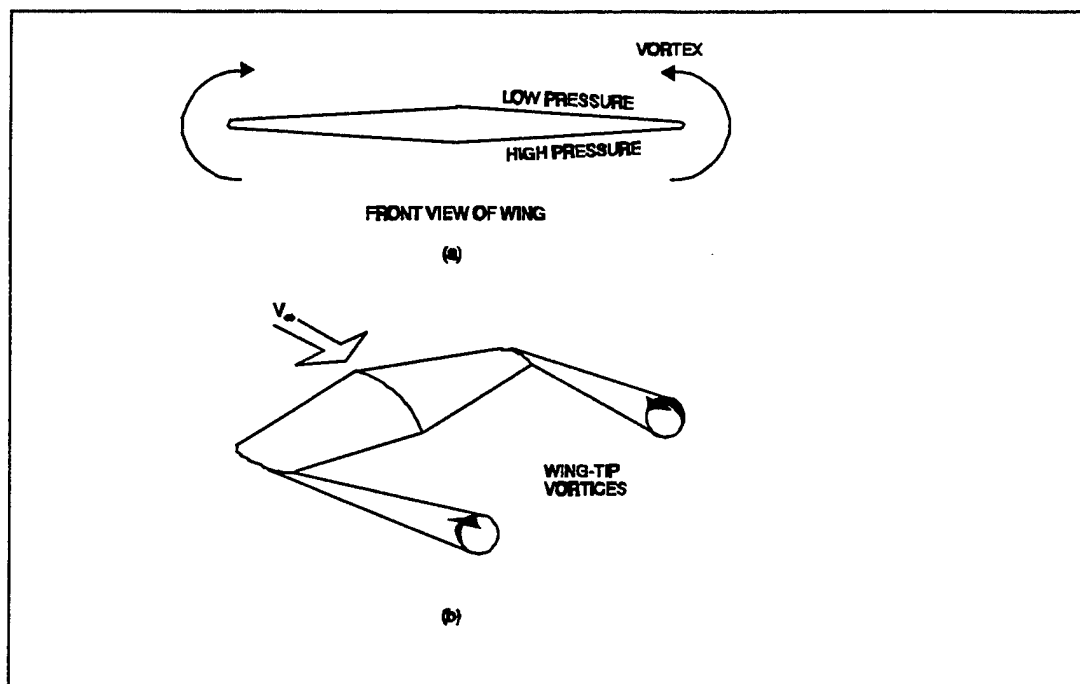
FIGURE 2.36   ORIGIN OF WINGTIP VORTICES ON A FINITE WING (Ref. 1)

On the lower surface, the pressure is relatively high in comparison to that on the upper. The net result of this pressure differential is for the high pressure flow to "seek" the low pressure region--the result is a vortex trailing from each wing tip. These small "tornados" create a downwash (downward component of velocity) in the vicinity of the wing. Ultimately this downwash influences the lift and drag characteristics of the wing.

2.7.3 Calculation of Induced Drag

As discussed in the previous section, a downwash is created near the wing. This, in turn, alters the flowfield as "seen" by the wing. Consider the following sketches:
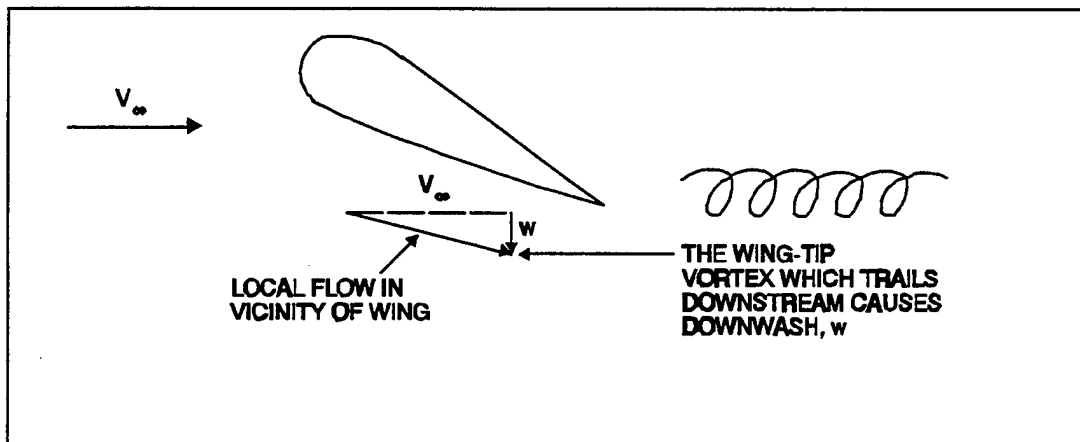
FIGURE 2.38 THE ORIGIN OF DOWNWASH (Ref. 1)
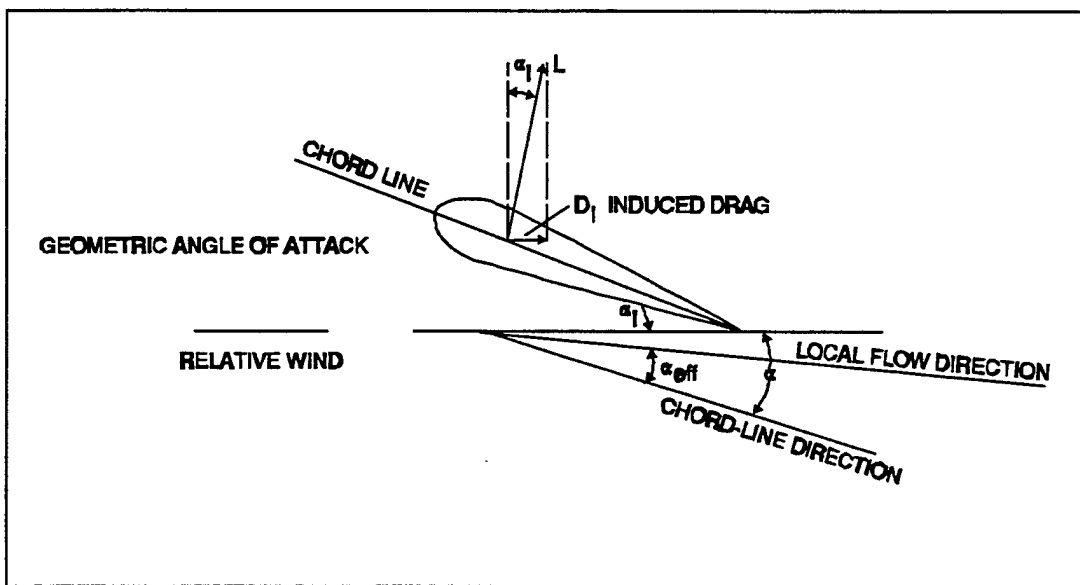
FIGURE 2.39 THE ORIGIN OF INDUCED DRAG (Ref. 1)

$$\sin \alpha_i = \frac{D_i}{L}$$

$$\alpha_i = \frac{C_L}{(\pi)(AR)} \quad (for\ constant\ downwash)$$

Assume α_i is small, therefore: $\sin \alpha_i = \alpha_i$

$$C_{D,i} = \frac{D_i}{q_\infty S} = \frac{L \sin \alpha_i}{q_\infty S} = \frac{C_L q_\infty S (C_L/(\pi)(AR))}{q_\infty S} = \frac{C_L^2}{\pi e (AR)}$$

A few notes with regards to the above equation for the induced drag coefficient:

1. "e" is called the span efficiency factor. A value of 1.0 is optimum--it applies for the case of constant downwash along the span of the wing (obtained from an "elliptical lift distribution"). For other planforms the efficiency factor is on the order of 0.85.

2. A high aspect ratio wing reduces the effects of induced drag.

3. Induced drag coefficient is proportional to the lift coefficient, <u>squared</u>--at high angles of attack, the induced drag is very high! Does this make sense?

Pause again...finite wing geometry introduces a fourth form of drag--skin friction, pressure, and wave drag (if supersonic) still exist. In summary, the drag coefficient for a finite wing is written as follows:

$$C_D = C_d + \frac{C_L^2}{\pi e (AR)}$$

where $C_d = C_{d,f} + C_{d,p} + C_{d,w}$

Graphically, this information can be presented on a drag polar. Note the effect of decreasing the aspect ratio.
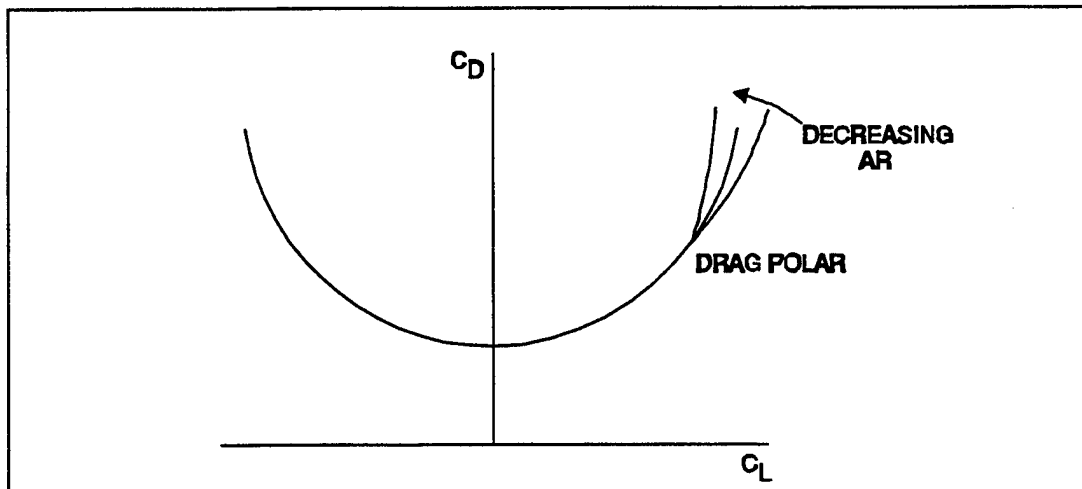
**FIGURE 2.40   SKETCH OF A DRAG POLAR, i.e., A PLOT OF
DRAG COEFFICIENT VS LIFT COEFFICIENT**

2.7.4 Lift Coefficient for a Finite Wing

What happens to the lift coefficient for the case of a finite wing, versus an airfoil? As
was done for the drag coefficient, we would like to modify airfoil data to adjust for the
effects of wing tip vortices. Qualitatively, the effect of aspect ratio on the lift curve
slope is shown below.
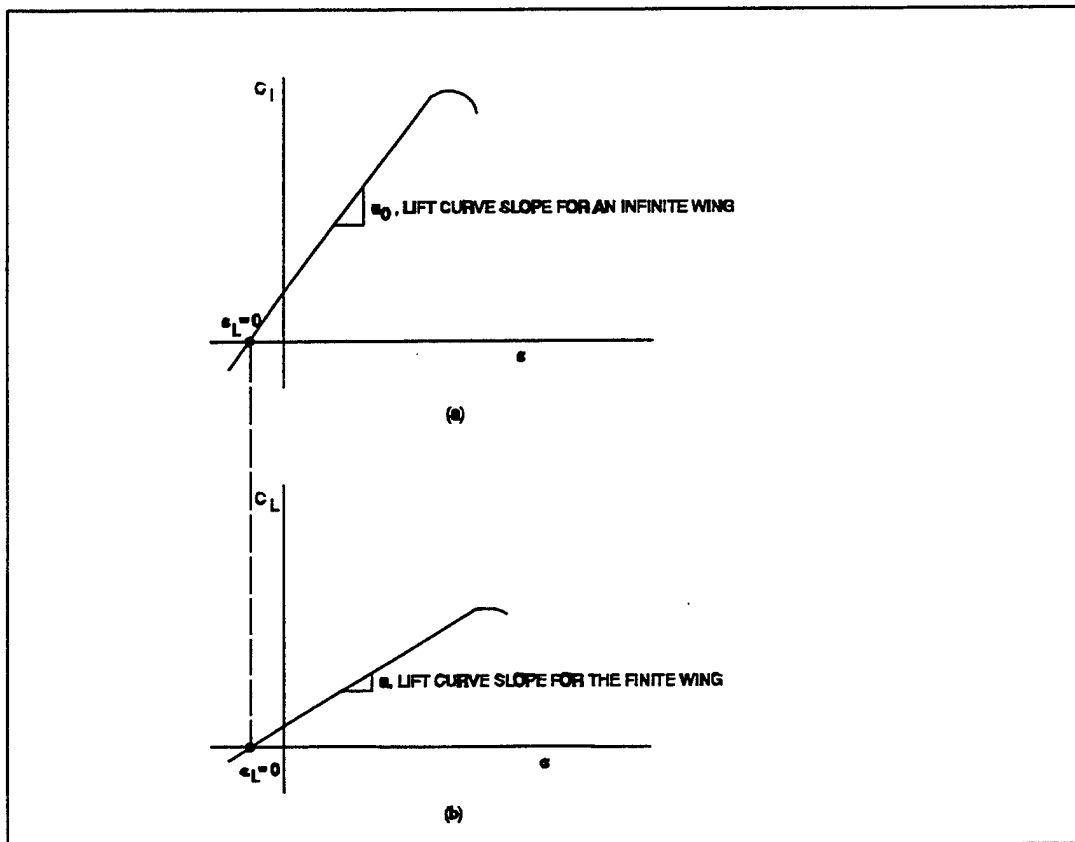
**FIGURE 2.41   DISTINCTION BETWEEN THE LIFT CURVE SLOPES
FOR INFINITE AND FINITE WINGS**

The following are the key points on this figure:

    1. As aspect ratio decreases, the lift curve slope (a, without a subscript) decreases.

    2. Note that the angle of attack at zero lift is the same value. Does this make sense?

The lift coefficient can be calculated from the following equation--it takes into account the effects of a decrease in lift curve slope.

$$C_L = a (\alpha - \alpha_{L=0})$$

$$where\ a\ =\ \frac{a_o}{1 + \dfrac{57.3 a_o}{\pi e (AR)}}$$

2.8 SWEPT WINGS AND HIGH LIFT DEVICES

2.8.1 Swept Wings

In order to delay the effects of drag divergence, wing sweep is typically used. The following shows the effect of a swept wing on the critical Mach number. If the critical Mach number can be increased, likewise will the drag divergence Mach number.
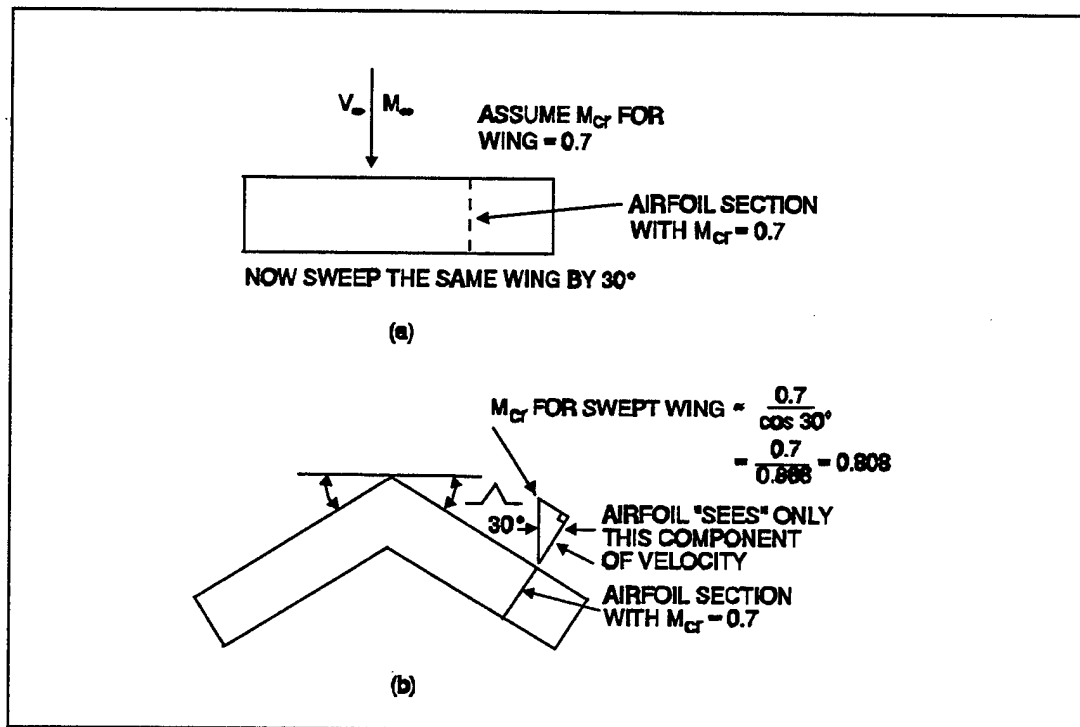
FIGURE 2.42   EFFECT OF A SWEPT WING ON CRITICAL MACH
                        NUMBER (Ref. 1)

The following equation is used above to determine the effect of wing sweep on the

critical Mach number:

$$M_{cr_{(swept)}} = \frac{M_{cr_{(unswept)}}}{\cos\Lambda}$$

In the above case, the critical Mach number was raised from 0.7 to 0.808 by sweeping the wing 30 degrees. The flow only "sees" the component of velocity perpendicular to the leading edge--this component governs the flow over the wing. Does this also apply for the case of forward swept wings? What is another way to delay the effects of drag divergence?

2.8.2 Stalling Speed

The slowest speed an airplane can fly in straight and level flight is called the "stalling speed" (V_{stall}). The equation for stalling speed is derived below:

$$C_L = \frac{L}{q_\infty S} = \frac{L}{\frac{1}{2}\rho_\infty V_\infty^2 S} = \frac{W}{\frac{1}{2}\rho_\infty V_\infty^2 S}$$

Solving for velocity and letting C_L equal $C_{L_{max}}$:

$$V_{stall} = \sqrt{\frac{2W}{\rho_\infty S\, C_{L_{max}}}}$$

Stalling speed can also be expressed in terms of equivalent airspeed, as shown below:

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2 = \frac{1}{2}\rho_\infty \left[V_e \sqrt{\frac{\rho_{SL}}{\rho_\infty}} \right]^2$$

or: $$V_{e_{STALL}} = \sqrt{\frac{2W}{\rho_{SL}\, S\, C_{L_{max}}}}$$

$$= \frac{1}{2}\rho_{SL} V_e^2$$

From this equation, the following observations can be made:

1. In order to decrease the stalling airspeed, a high max lift coefficient is desirable.

2. An increase in the planform area (S) will decrease the stalling airspeed.

3. Recall that the maximum lift coefficient is a function of Reynolds Number. All else constant, an increase in altitude (decrease in density) will result in a decrease in Reynolds Number...therefore, the max lift coefficient goes down with altitude. The aircraft stalls at a higher equivalent (or true) airspeed.

2.8.3 High Lift Devices

Basically, high lift devices are used to increase C_{Lmax}. Maneuvering, slow sped flight, and landing require high lift coefficients. In order to provide lift coefficients greater than the maximum lift coefficient of a given airfoil, it is necessary to resort to special hardware. Devices of this type are slots, slats, boundary layer control (BLC), and flaps (both leading and trailing edge). Some of these devices are characteristically low speed devices, i.e., slots and BLC, while others are suitable for both high and low speed applications, i.e., flaps and slats. Numerous variations of these devices have been proposed and used on operational aircraft. Each type will be defined and their individual effects on an aircraft discussed briefly.

Basically, increasing the total lift of a wing can be accomplished by any one or combination of three methods. The first would be increase the wing area, the second, increase the camber of the wing, and the third would be to delay separation through some means of boundary layer control

2.8.3.1 Flaps

Flaps are high lift devices which are basically hinged leading or training edge wing sections. They increase lift by increasing the camber or area of the wing.

2.8.3.1.1 Trailing Edge Flaps

Trailing edge flaps are normally 15 to 25% of the chord; although they can be found up to 40% chord.

Several arrangements are commonly used. The basic types of trailing edge flaps are shown in Figure 2.43.
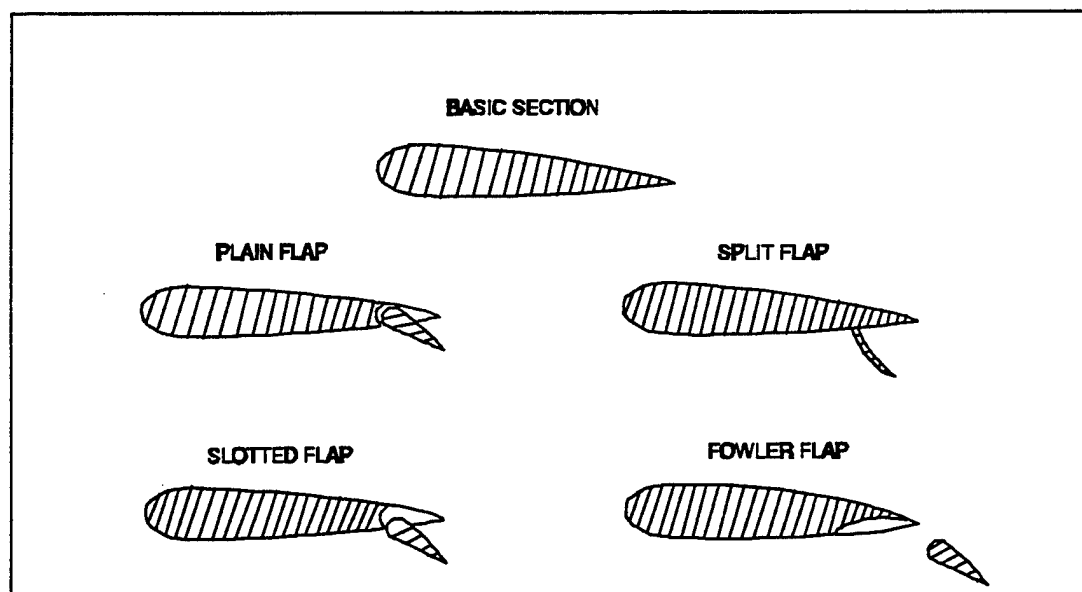
FIGURE 2.43   BASIC TYPES OF TRAILING EDGE FLAPS
(Ref. 7)

The plain (or simple) flap shown in Figure 2.43 is a simple hinged portion of the wing trailing edge. The split flap is essentially a plate deflected from the lower surface of the wing.

The slotted flap is similar to the plain flap, but the gap between the main wing section and flap leading edge is given a specific contour so that high energy air from the lower surface is ducted to the flap upper surface when the flap is deflected. The high energy air from the slot re-energizes the upper surface boundary layer and delays separation.

The Fowler flap arrangement is similar to the slotted flap; however, when extended it lowers and translates aft, thus increasing wing area as well as camber. Because of the unique movement of this type of flap, the mechanism is quite heavy and complicated and therefore may not be practical for certain aircraft applications.

Figure 2.44 shows that all four basic trailing edge flap types provide a significant increase in $C_{l_{max}}$. As expected, the more complicated the flap arrangement, the larger the increase in $C_{l_{max}}$. Note that addition of flaps does not change the lift curve slope but that the lift curve shifts parallel to itself for all four basic flap types shown. Therefore, any required value or lift coefficient occurs at a lower angle of attack.
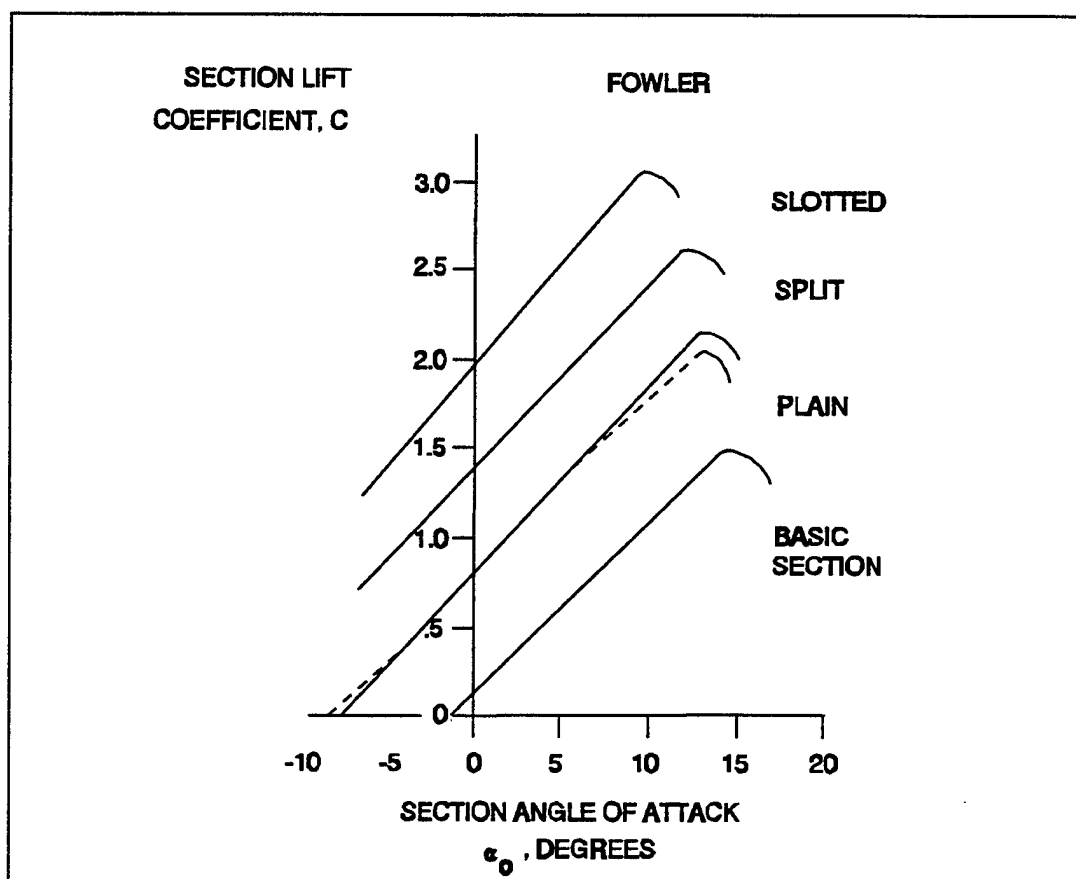
FIGURE 2.44   EFFECT OF TRAILING EDGE FLAPS ON THE LIFT CURVE (Ref. 7)

The effectiveness of flaps on a wing configuration depends on many factors. One important factor is the amount of wing area affected by the flaps. Since a certain amount of wingspan is normally reserved for ailerons, the actual wing maximum lift properties will be less than that of the flapped two-dimensional section. Recently attempts (some more successful than others) have been made to increase total aircraft lift coefficient by using full-span flaps in conjunction with spoilers for roll control.

2.8.3.2 Leading Edge Flaps

There are many variations in the design of leading edge flaps. Many leading edge flaps also form leading edge slots when extended. Figure 2.45 illustrates several types of leading edge flap devices.
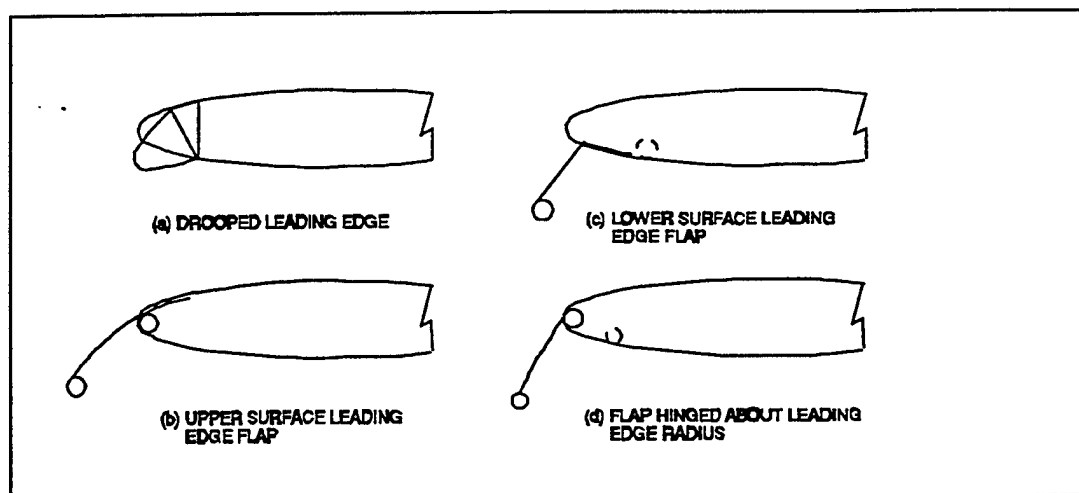
FIGURE 2.45   VARIOUS LEADING EDGE FLAP DEVICES (Ref. 8)

A currently popular leading edge flap which pivots about the leading edge of the airfoil was introduced in 1942 and is referred to as a "Krueger flap". Figure 2.46 is an illustration of a Krueger flap currently in operation. The obvious complexity of this device serves to point out that use of certain types of high lift devices is necessarily restrictive in application.
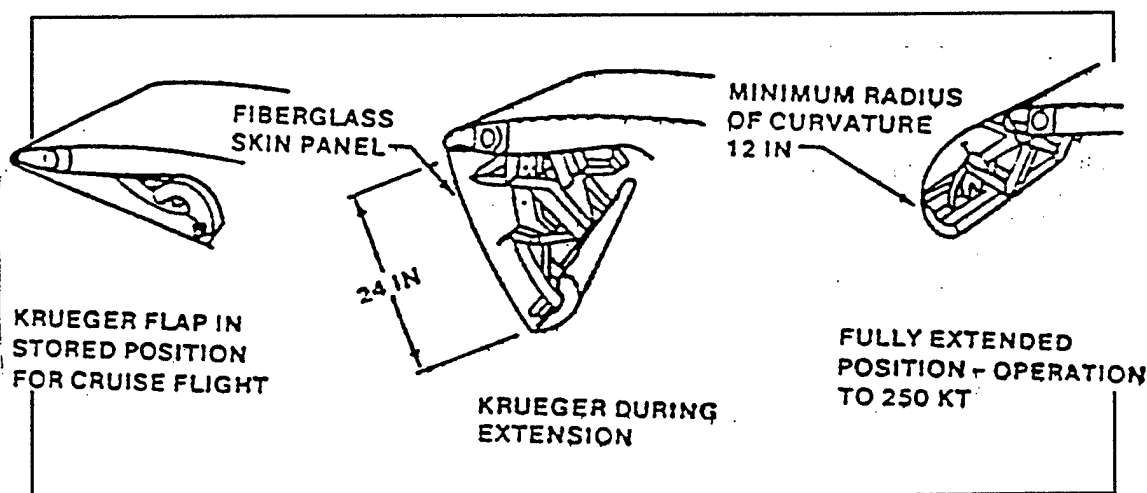
FIGURE 2.46   B-747 VARIABLE CAMBER LEADING EDGE KRUEGER FLAP (Ref. 8)

The combination of a slotted-Krueger and a multi-sectioned-Flowler flap is shown in Figure 2.47.
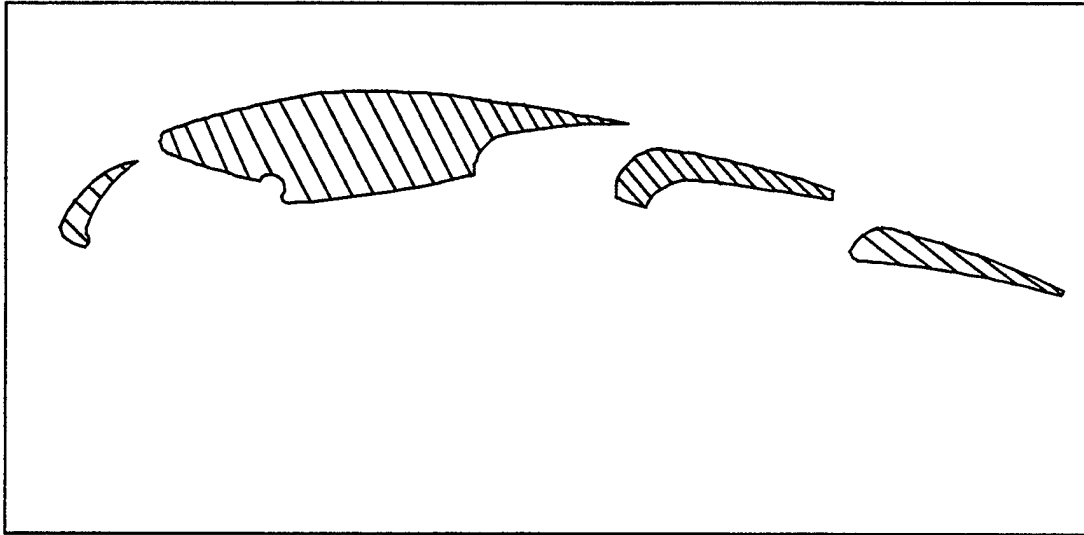
FIGURE 2.47   KRUEGER-FOWLER FLAP CONFIGURATION

2.8.3.3 Boundary Layer Control (BLC)

Slots and slats are aerodynamic means of affecting boundary layer control. Boundary layer control may also be accomplished by artificial means such as blowing air provided by a compressor over the wing or by drawing the low energy boundary layer through the surface of the wing by some suction device.

2.8.3.3.1 Slot

A fixed slot is in effect an aerodynamic boundary layer control device since it takes high energy air from the lower surface of the wing and ducts it through the wing into the low energy boundary layer on the upper surface, as shown in Figure 2.48.
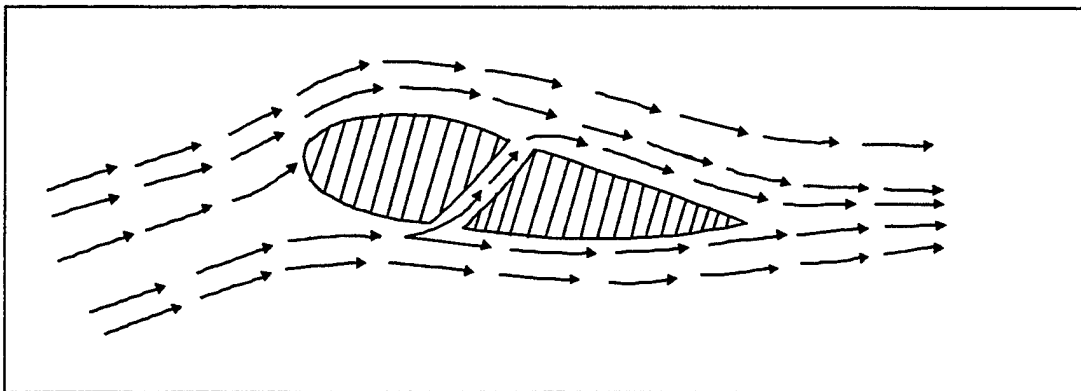
FIGURE 2.48   FIXED SLOT

In doing so, it delays separation and allows higher lift coefficients to be developed. A slot is relatively ineffective at low angles of attack but becomes very effective at high angles, thus improving the high lift characteristics without significantly compromising the low lift characteristics. Slots are used only on very slow speed aircraft since they cause very high drag at higher speeds.

2.8.3.3.2 Slat

A slat operates on the same principle as a slot except that it is located near the leading edge and acts like an additional airfoil in front of the basic airfoil. Its function is to direct air flow over the leading edge of the airfoil, as shown in Figure 2.49. Fixed slats also cause high drag at high speed.
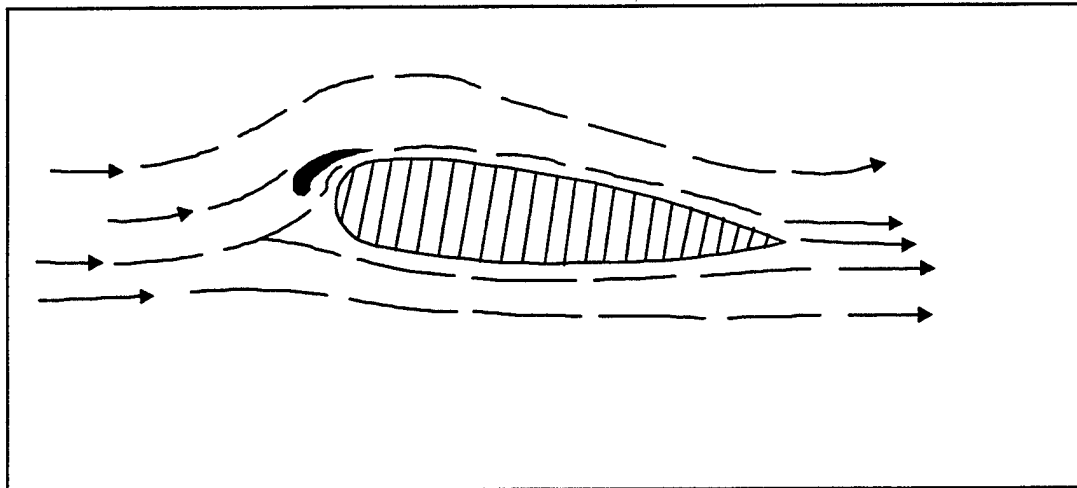
FIGURE 2.49   FIXED SLAT

A slat may also be of the moveable type which remains retracted at high speed and extends at low speed and high angles of attack, as shown in Figure 2.50. This increases the wing area slightly as well as increasing the flow over the upper surface of the wing.
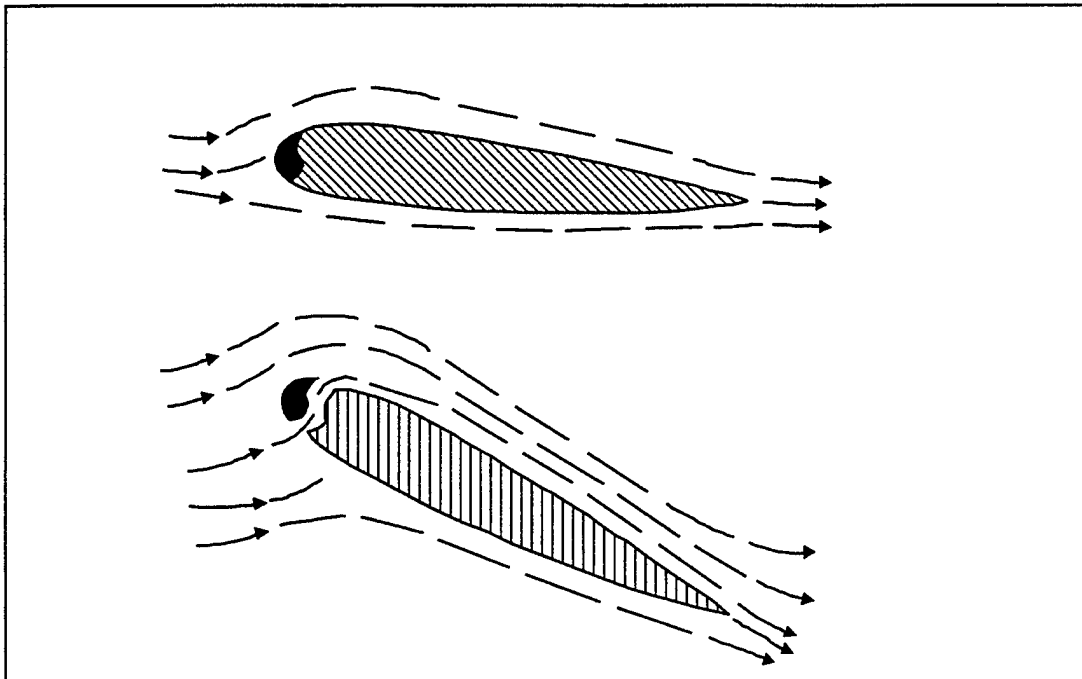
FIGURE 2.50   MOVABLE SLAT

The slot or extended slat simply delays stall to a higher angle of attack as shown in Figure 2.51.
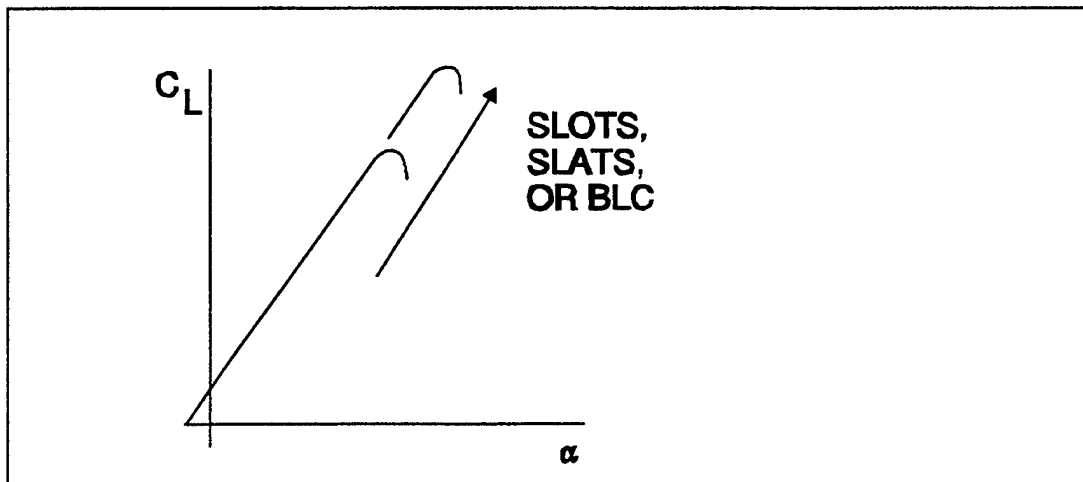
FIGURE 2.51   EFFECT OF SLOTS, SLATS OR BLC ON THE LIFT CURVE

2.8.3.4 Blowing and Suction

Figure 2.52 illustrates boundary layer control techniques.
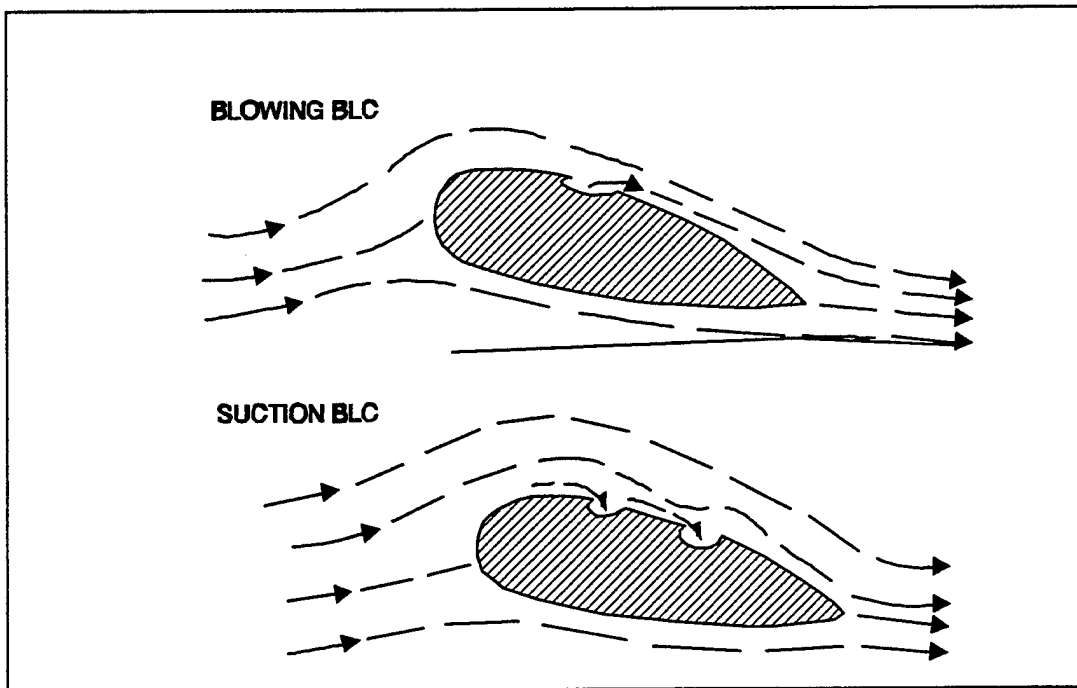
FIGURE 2.52   BOUNDARY LAYER CONTROL

Boundary layer control has exactly the same effect on the lift curve as does the use of slots or slats as shown in Figure 2.51 Almost any combination of slots, slats, flaps and BLC can be found today in modern aircraft designs. Slots or extended slats used alone increase maximum lift coefficient; however, they can produce undesirable high angles of attack at low speeds as shown in Figure 2.53. For this reason, slots or slats are usually used in conjunction with flaps since flaps provide a reduction in the maximum lift coefficient angle of attack. Figure 2.53 summarizes various high lift devices used together.
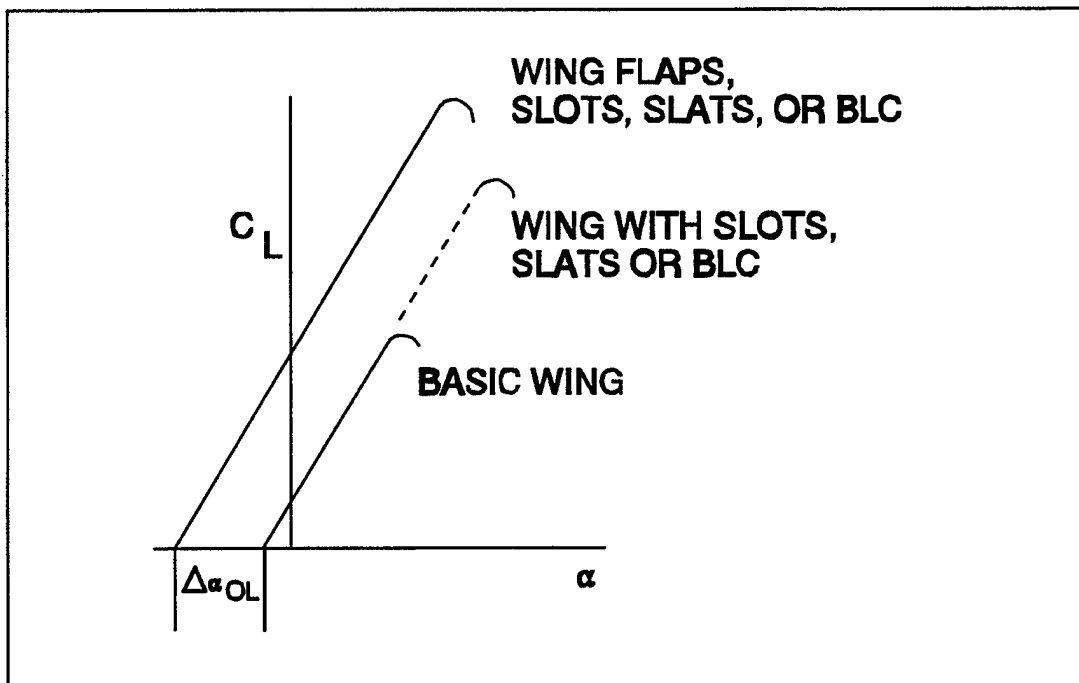
FIGURE 2.53   HIGH LIFT DEVICES

Figure 2.54 shows the maximum sectional lift coefficient generated by the various high lift devices on a typical NACA airfoil.
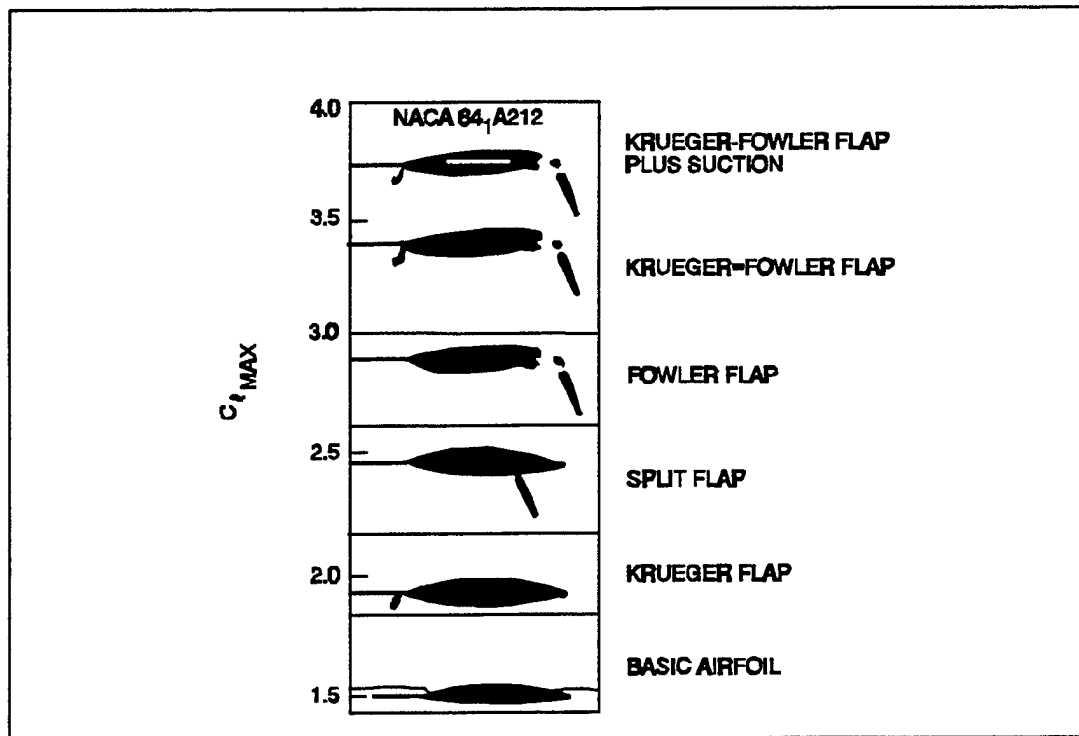
FIGURE 2.54   MAXIMUM SECTIONAL LIFE COEFFICIENTS

2.8.3.5 Vortex Generators

Although they are not generally considered high lift devices, vortex generators also prevent separation aerodynamically. As shown in Figure 2.55, these devices are rows of small vanes placed perpendicular to the surface of an aircraft, each acting like a small wing whose wingtip vortex brings high energy air from outside the boundary layer down into the boundary layer, thus re-energizing it.
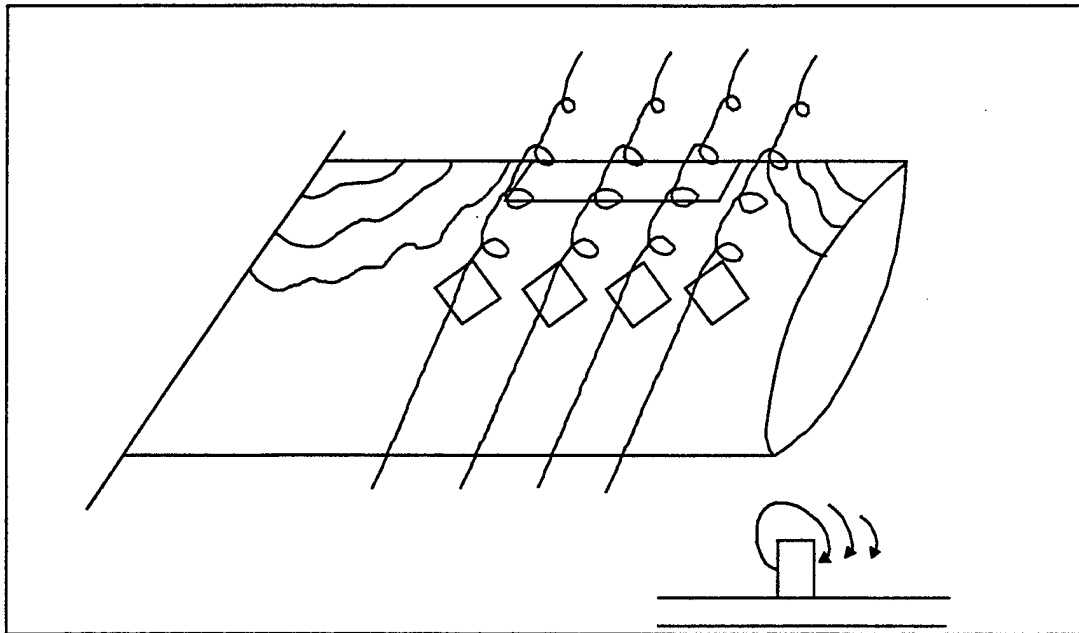
FIGURE 2.55   VORTEX GENERATORS

Vortex generators normally add little toward increasing maximum lift coefficient. They are generally used to prevent local areas of separation such as in front of an aileron, which causes the flow over the aileron to stay attached to a higher angle of attack than the rest of the wing. The "strategic" placement of vortex generators is, in general, an art rather than a science.

2.9 AIRCRAFT PERFORMANCE I

2.9.1 Introduction

To this point, we have considered only airfoils and finite wings in our discussion of aerodynamics. With the fundamentals in place, we can now look at the complete airplane.

2.9.2 Lift Coefficient

We have defined the lift coefficient in previous discussions. Now, however, we concern ourselves with the lift coefficient for the entire airplane--this includes the contributions from not only the wing, but also the fuselage, horizontal tail, strakes,

etc. The lift coefficient is defined as:

$$C_L = \frac{L}{q_\infty S}$$

where S is the planform area of the wing.

The "load factor" (n), or "cockpit g's", is defined as the ratio of lift over weight:

n = L / W

Having introduced load factor, the lift coefficient can be written as:

$$C_L = \frac{L}{q_\infty S} = \frac{nW}{q_\infty S} = \frac{nW}{\frac{1}{2}\rho_{SL} V_e^2 S}$$

$$\rho_{SL} = 0.002377 \frac{slugs}{ft^3}$$

$$therefore: \quad C_L = \frac{841.5nW}{V_e^2 S}$$

or

$$C_L = \frac{nW}{\frac{1}{2}\rho_\infty V_\infty^2 S}$$

$$M_\infty = \frac{V_\infty}{a_\infty} = \frac{V_\infty}{\sqrt{\gamma R T_\infty}}$$

therefore: $V_\infty^2 = M_\infty^2 (\gamma R T_\infty)$

$$\rho_\infty = \frac{P_\infty}{R T_\infty}$$

or

$$C_L = \frac{nW}{\dfrac{1}{2}\left(\dfrac{P_\infty}{RT_\infty}\right)\left(M_\infty^2 \gamma RT_\infty\right)S} = \frac{nW}{1481\delta M_\infty^2 S}$$

$$where \;\; \delta = \frac{P}{P_{SL}}$$

2.9.3 Total Aircraft Drag

2.9.3.1 The Drag Polar

The total drag on an aircraft can be classified as follows:
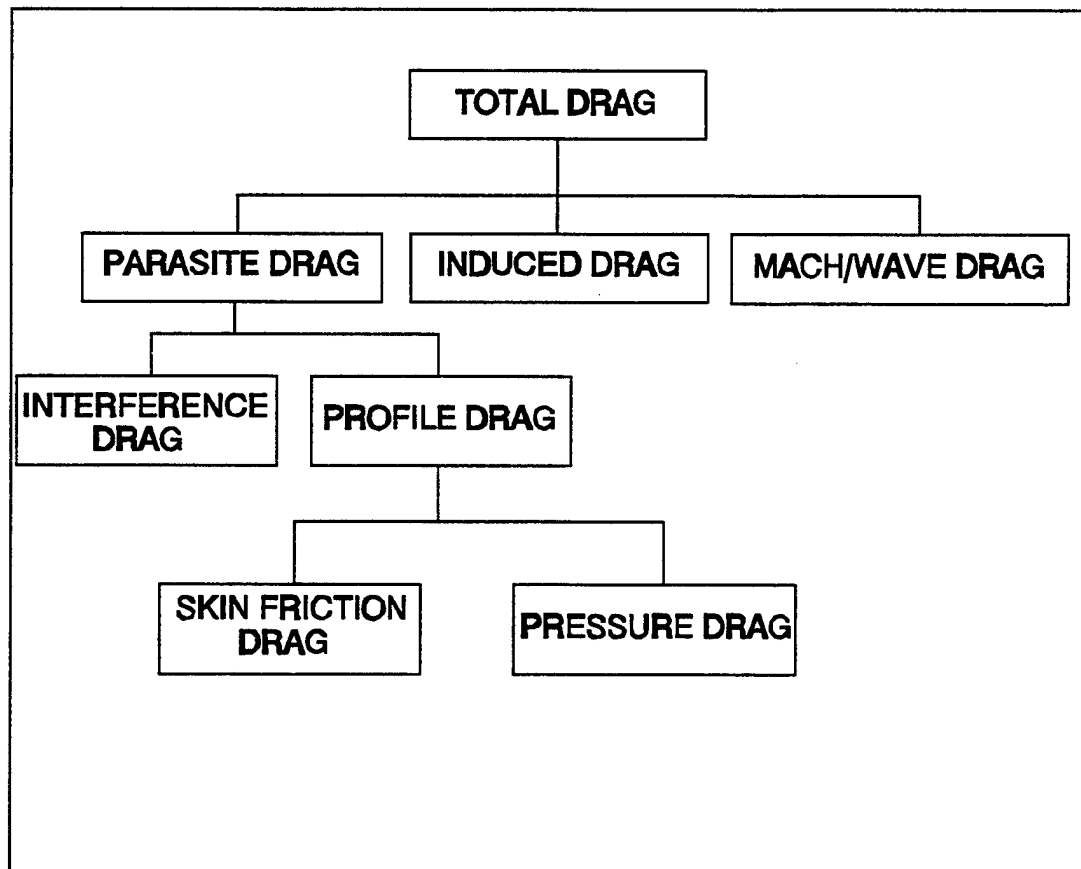
FIGURE 2.56   DRAG CLASSIFICATION

Typically, the aerodynamics of an airplane is presented in the form of a drag polar--this can be done in the form of an equation, a graph, or both. The total airplane drag coefficient (or drag polar), presented as an equation, is:

$$C_D = C_{D,o} + \frac{C_L^2}{\pi e AR}$$

$$= C_{D,o} + K\ C_L^2$$

$$where\ K = \frac{1}{\pi e AR}$$

The FIRST term (right hand side) is called the "parasite" or "zero lift drag coefficient". The parasite drag coefficient is a constant, for a specific aircraft configuration--it is not a function of lift. Included are profile drag (skin friction and zero lift pressure drag) and interference (more about parasite drag later).

The SECOND term is called the "drag due to lift" coefficient. Included in this are the induced drag on all lifting surfaces and the increment of parasite drag when the aircraft is generating lift. "e" is called the "Oswald efficiency factor" (on the order of about 0.8)--this is a "fudge factor" (usually determined through wind tunnel/flight testing) that takes into account such effects as a nonelliptical lift distribution and the variation of parasite drag with lift.

Additionally, for flight in excess of the critical Mach number, wave drag is another contributing factor--the drag coefficient is modified as such:

$$C_D = C_{D,o} + \frac{C_L^2}{\pi e AR} + C_{D,w}$$

It is very convenient to present the drag polar graphically. Typically, this is done in two ways:
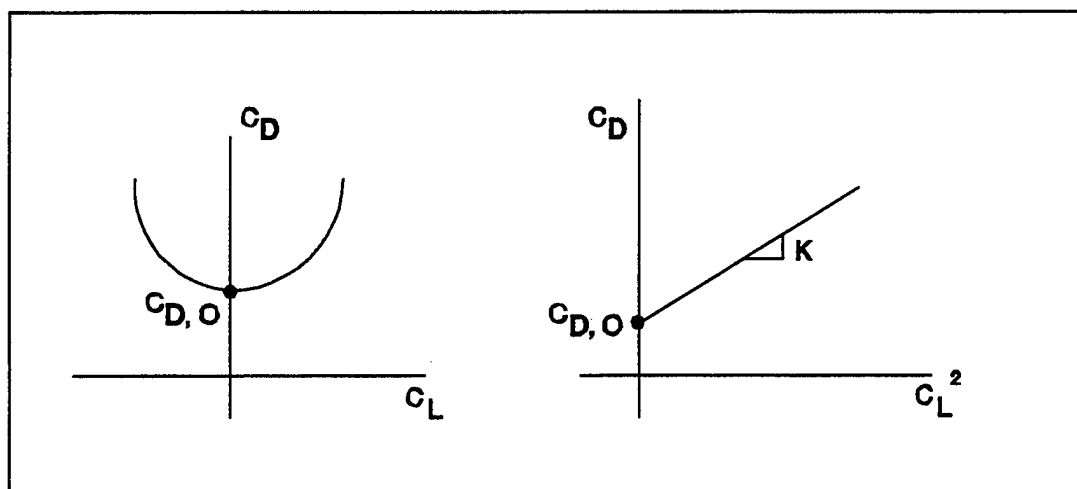
FIGURE 2.57 TYPICAL DRAG POLARS

2.9.3.2 Interference Drag

Interference drag is generated when several objects are placed in the same airstream creating eddy currents, turbulence or restrictions to smooth flow. For example, the air flowing along the fuselage collides with the air flowing over the wing in the area of the wing root. The effects of this collision can be reduced by allowing a smoother merging of the two air currents by installation of a fairing at the fuselage-wing root junction as shown in Figure 2.58. If an external store is hung on the wing of an aircraft and the drag of the aircraft and that of the store are known, the drag actually produced is greater than the sum of the drag of the individual components because of the interference drag created.

Any time two parts of an aircraft are joined or any object is placed on or in close proximity to an aircraft, interference drag is created. In some cases, however, overall drag can actually be reduced by proper streamlining of the flow. An example of this beneficial effect is evidenced in the addition of the conformal tanks on the F-15.
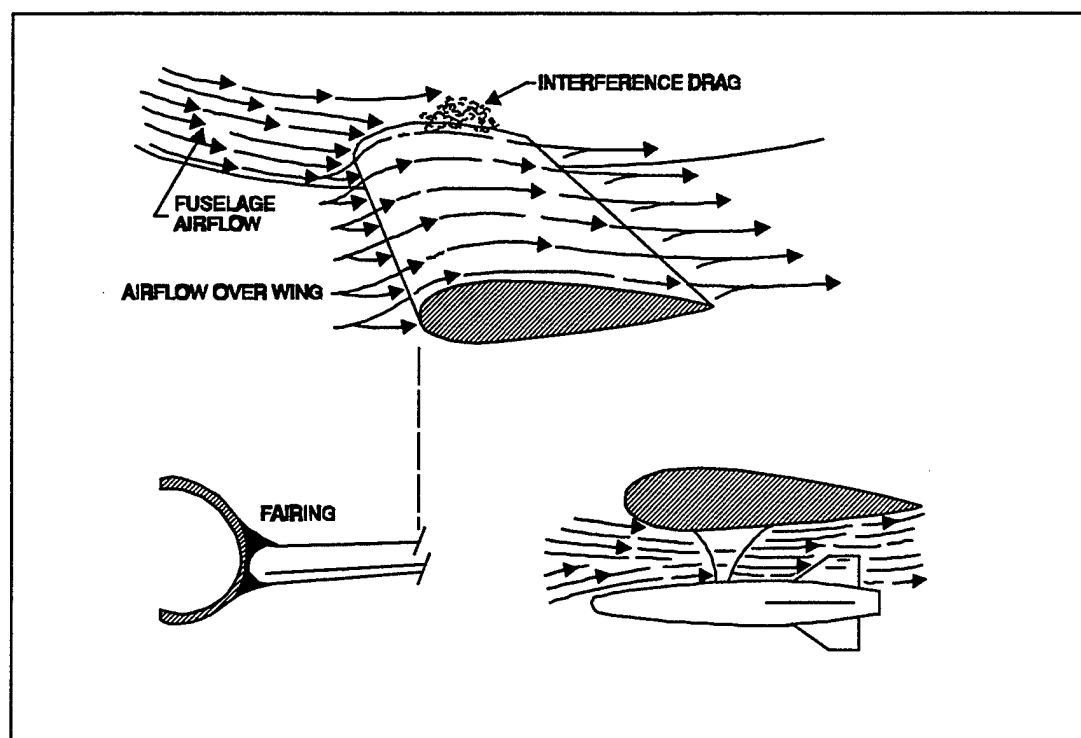
FIGURE 2.58   INTERFERENCE DRAG (Ref. 9)

2.9.3.3 Parasite Drag

Parasite drag is the sum of profile and interference drag on the wingbody combination. Note that profile drag in this case is the skin friction and pressure drag of the entire wing body combination. The drag equation can be written:

$$D_p = C_{D,o} \; q_\infty S$$

where $C_{D,o}$ is the parasite drag coefficient

Since an aerodynamic coefficient is completely arbitrary, it may be based on any area. Conventionally, the standard area is the wing area. However, for such external stores

as bombs, drop tanks and missile launchers, the projected frontal area is frequently used.

To determine the increment in drag coefficient that has to be added to an aircraft's drag coefficient due to carrying an external store, the relationship in below is used.

$$\text{Drag}_{\text{store}} = \Delta\text{Drag}_{\text{aircraft}}$$

$$C_{D_x} q S_\pi = \Delta C_{D,o} \, qS$$

from which a parasite drag increment can be determined.

$$\Delta C_{D,o} = \frac{C_{D_x} S_\pi}{S}$$

where S is the wing area, S_π is the store's projected frontal area, and $C_{D,\pi}$ is the store's proper drag coefficient (see, for example, Table 2.2).

2.9.3.3.1 Drag Counts

Since it is awkward to speak of increments of drag coefficient based on wing area, it is customary to use the expression "counts of drag" where one count of drag is a drag coefficient of 0.0001. Thus, a rocket launcher which increases the airplane's parasite drag coefficient by 0.0010 would be said to have ten counts of drag. The term "drag index" is closely related to this. A drag index of ten, for example, means that this configuration has ten counts of drag more than the basic configuration of the airplane. Aircraft flight manuals often use the drag count system in presenting performance data.

2.9.3.3.2 Equivalent Flat Plate Area

Instead of indicating the aerodynamic characteristics of a bomb or an aircraft by a parasite drag coefficient, frequently an "equivalent flat plate area" is used. This means that the bomb could be replaced mathematically by a flat plate having the same drag as defined below.

$$D_{P_{plate}} = C_{D_{plate}} q_\infty S_{plate}$$

where

$$C_{D_{plate}} = 1.28 \text{ (determined from wind tunnel testing)}.$$

Therefore

$$C_{D,0} \, q_\infty S = 1.28 \, q_\infty \, S_{plate}$$

or

$$S_{plate} = \frac{C_{D,o} S}{1.28}$$

The equivalent flat plate area is a convenient and graphic way of expressing the parasite drag of a body. A drag coefficient of 0.003 has little significance to someone unfamiliar with the normal magnitude of bomb coefficients. An equivalent flat plate area of one square foot has a very definite physical significance. Often, wind tunnel external store testing data are presented in terms of equivalent flat plate area. Sometimes aircraft are compared aerodynamically by computing their equivalent flat plate areas.

2.9.3.4 Miscellaneous Forms of Drag

The following definitions and explanations are offered for various other terms used to describe types of drag.

2.9.3.4.1 Ram Drag

Ram drag is drag due to ram compression in the diffuser of a turbojet or turbofan engine. This term is widely used in propulsion. Net thrust is usually defined as gross thrust minus ram drag.

2.9.3.4.2 Cooling Drag

Cooling drag is drag due to energy loss when air is forced past cylinders on an air cooled reciprocating engine. This is a major source of parasite drag on reciprocating propeller driven aircraft. Much flight test time is usually devoted to minimizing cooling drag while providing adequate engine cooling.

2.9.3.4.3 Trim Drag

Trim drag is an additional drag force which results from the use of the horizontal tail in trimming the aircraft, i.e., in maintaining longitudinal equilibrium. At high speeds, the tail usually carries a download, which means that the wing must provide additional lift. Therefore, the drag also increases, and the necessary increase in incidence also causes critical Mach to be reduced. This effect is particularly important

at high altitude, and if the tail moment arm is short. Tailless aircraft also suffer from high trim drag. One of the advantages of the Canard configuration is that, with the -tail in front of the wing instead of behind it, it carries an upload for trim at high speeds. This is equivalent to providing negative trim drag.

2.9.3.5 Variables Affecting the Drag Polar

2.9.3.5.1 Variables Affecting Drag Coefficient

We know from dimensional analysis that

$$C_D = f(M, R_e, \alpha)$$

and that geometrically similar shapes compared at the same Mach, Reynolds number and angle of attack have the same drag coefficient. The basic drag polar illustrates variation of drag coefficient with angle of attack in the guise of lift coefficient. Variations of Mach and Reynolds number will produce changes in the basic polar. Drag coefficient is also affected by two planform variables, aspect ratio and efficiency factor. The following discussion will illustrate the effects of all these variables on the basic drag polar.

2.9.3.5.2 Drag Polar Variation with Mach

When the flight velocity is greater than the critical Mach, the drag coefficient increases. Below critical Mach, the low speed polar is unchanged. This effect is shown in Figure 2.59.
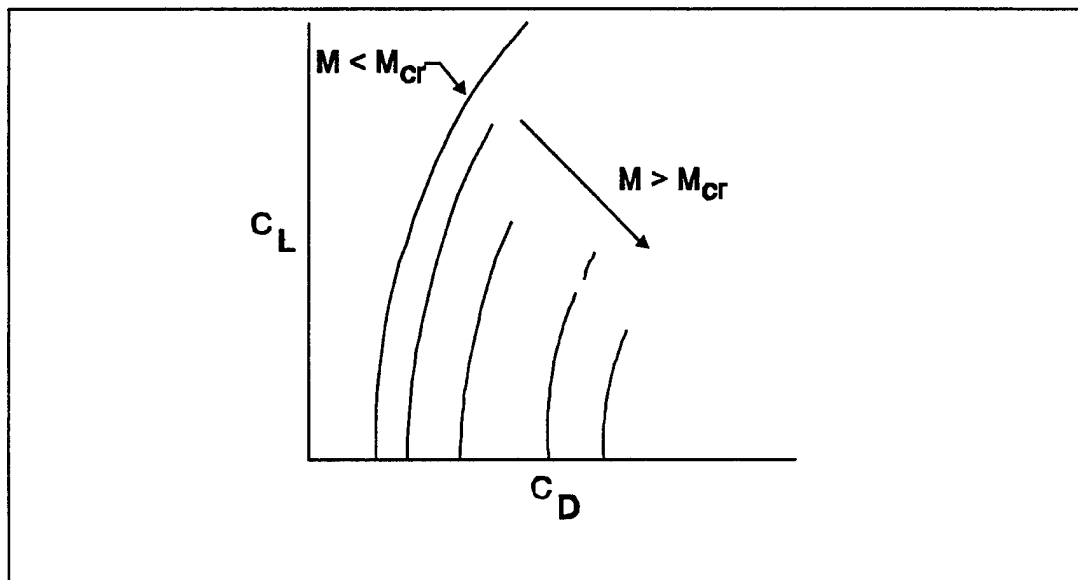
FIGURE 2.59   EFFECT OF MACH ON THE DRAG POLAR

Drag coefficient data are often presented versus Mach at constant values of angle of attack, or more commonly constant lift coefficient as shown in Figure 2.60. In plotting Figure 2.60, parasite drag coefficient and lift coefficient are constant, and Mach drag coefficient is zero until critical Mach is reached. Figure 2.60 shows no increase in C_D until after the critical Mach is exceeded, then a sudden increase in drag coefficient
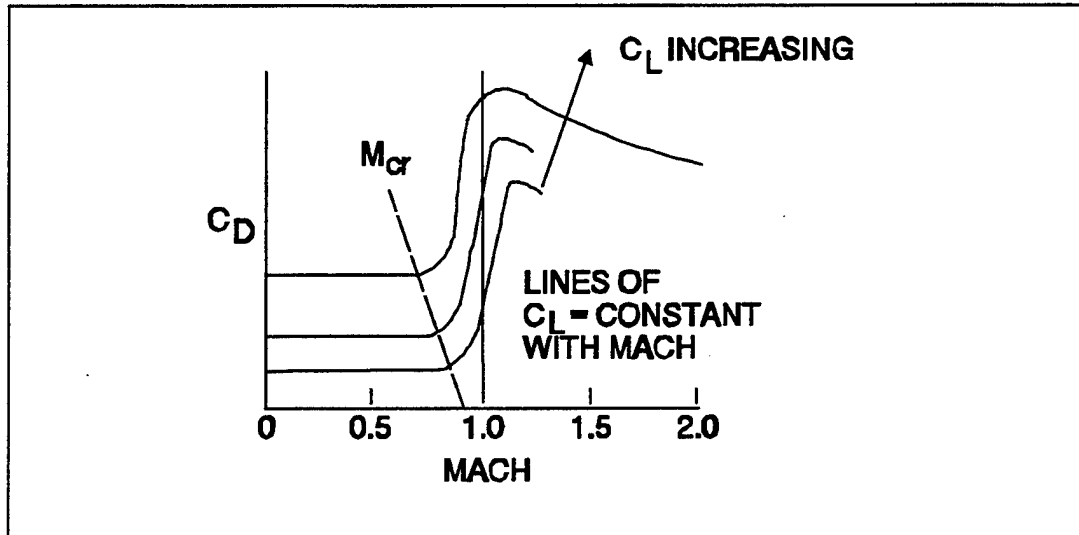
FIGURE 2.60   VARIATION OF DRAG COEFFICIENT WITH MACH AT CONSTANT LIFT COEFFICIENT

called drag divergence is experienced. This is caused by the formation of shock waves on the aircraft. The total drag coefficient must now include a term to account for wave drag.

$$C_D = C_{D,o} + \frac{C_L^2}{\pi e AR} + C_{D_w}$$

2.9.3.5.3 Drag Polar Variation with Reynolds Number

The primary result of increasing Reynolds number on the drag polar is to increase the available maximum lift coefficient, as shown in Figure 2.61.
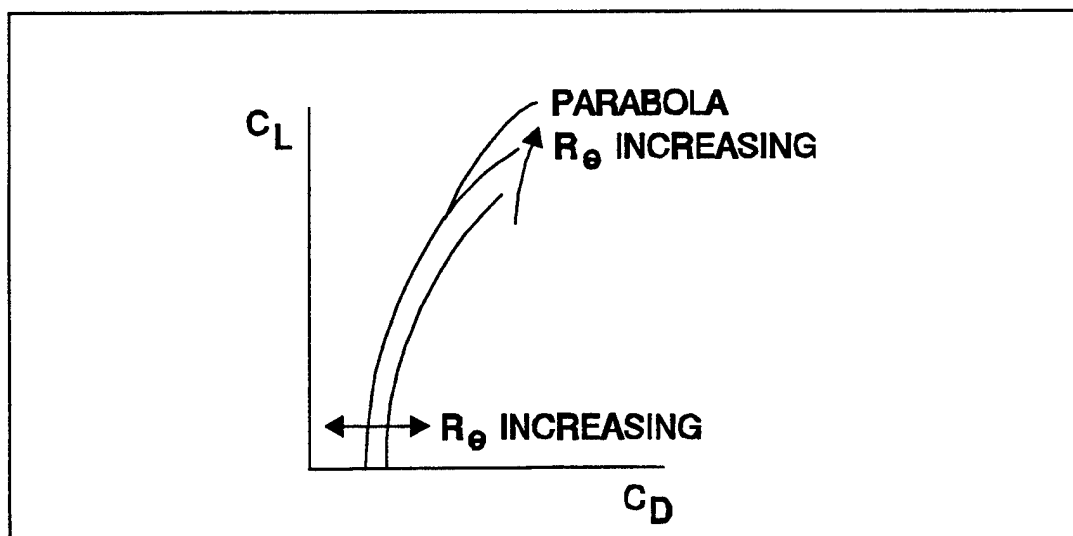
FIGURE 2.61   EFFECT OF REYNOLDS NUMBER ON THE DRAG POLAR

The previous discussion of Reynolds number effects on the lift curve also explains why the maximum C_L occurs at the highest Reynolds number on the drag polar. Increasing Reynolds number delays separation and allows the drag polar curve to remain parabolic to a higher angle of attack (lift coefficient). After separation, the total skin friction drag on the airfoil will decrease; however, pressure drag increases dramatically. Therefore, at a given lift coefficient at high angles of attack, total drag is lower at higher Reynolds number.

Since the Reynolds number of the flow describes how long the boundary layer will remain laminar before transitioning to turbulent flow, skin friction drag at lower angles of attack is also affected by changes in Reynolds number. Depending on the flow situation, the drag polar could show a decrease in $C_{D,O}$ as shown in Figure 2.61, or it could increase.

2.9.3.5.4 Drag Polar Variation with Oswald's Efficiency Factor
The following equation, which expresses the drag coefficient versus lift coefficient curve as a parabola, can be used to predict changes in the drag polar due to changes in Oswald's efficiency factor.

$$C_L^2 = \pi e AR \left(C_D - C_{D,o} \right)$$

The effort of changing Oswald's efficiency factor is shown in Figure 2.62.
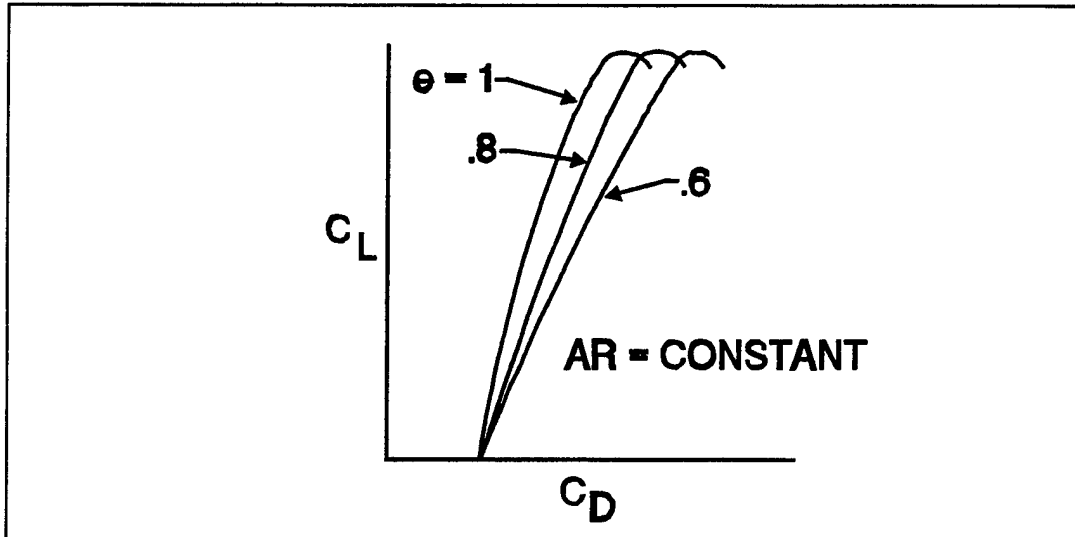
FIGURE 2.62   EFFECT OF OSWALD'S EFFICIENCY FACTOR ON THE DRAG POLAR

2.9.3.5.5 Drag Polar Variation with Aspect Ratio

The above equation (repeated below) can also be used to predict changes in the drag polar due to changes in aspect ratio.

$$C_L^2 = \pi e AR\left(C_D - C_{D,o}\right)$$

Increasing aspect ratio, AR, again causes the parabola to "open up" and results in larger values of lift coefficient for any given value of drag coefficient as shown in Figure 2.63.
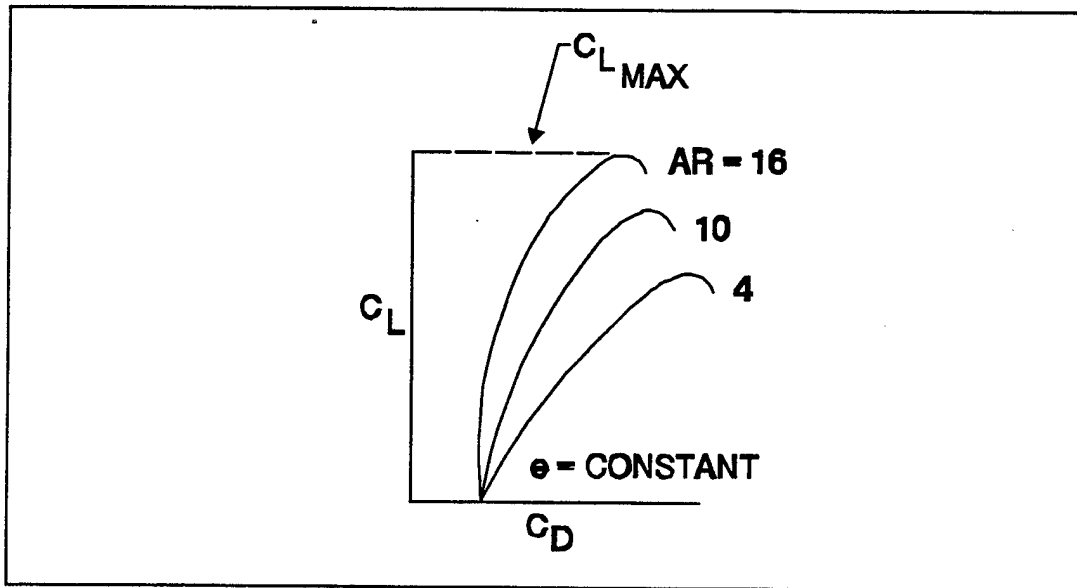
FIGURE 2.63   EFFECT OF ASPECT RATIO ON THE DRAG POLAR

2.9.3.5.6 Effect of Flaps on the Drag Polar

It is very difficult to make generalizations about the effect of flaps on the drag polar since flaps have such a wide variety of configurations as shown in Figure 2.64. In general, flaps increase parasite drag coefficient. They are usually installed to increase maximum lift coefficient. As can be seen in Figure 2.64, for some given values of lift coefficient, there are cases where total drag coefficient decreases with flap extension.
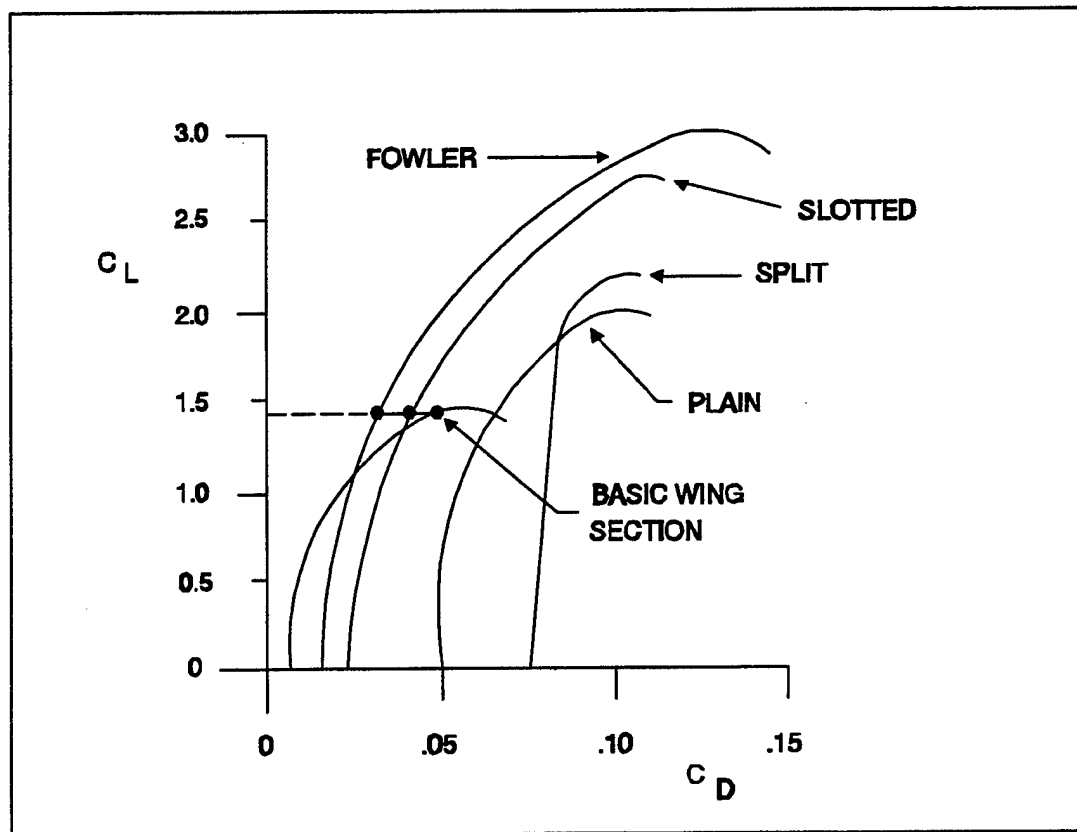
FIGURE 2.64   EFFECT OF FLAPS ON THE DRAG POLAR (Ref. 7)

Flaps are normally used in high angle of attack configurations such as takeoff and landing. Figure 2.65 illustrates the effect of flaps on the drag polar during flap extension or retraction. This figure points out that flap management should be exercised with care by the pilot due to large increases in drag during their extension.
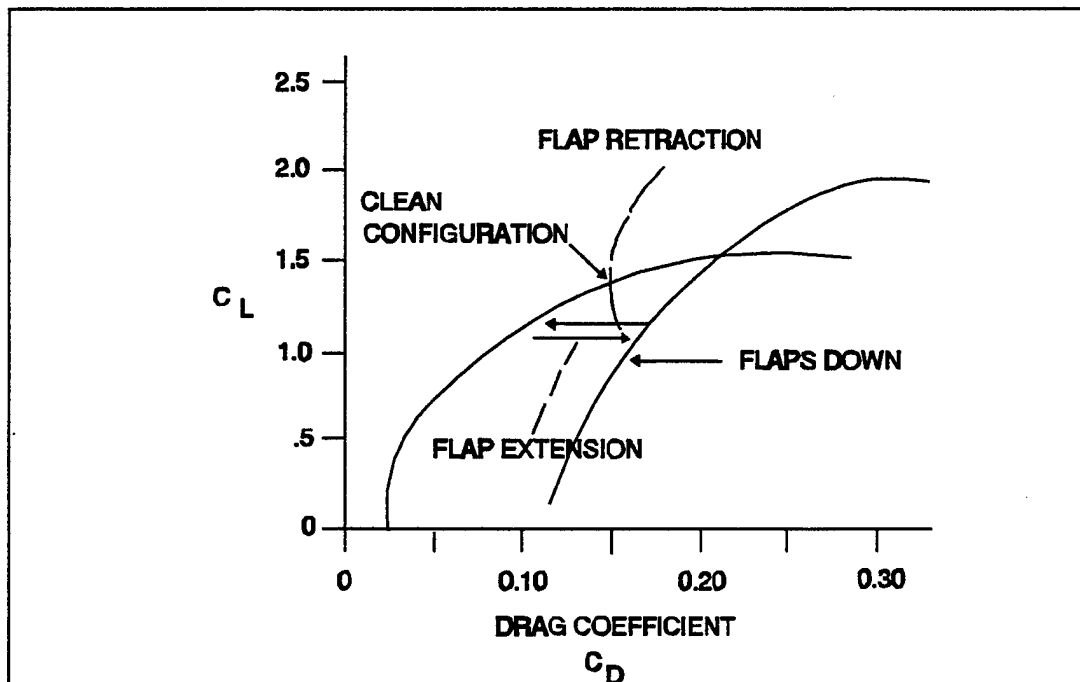
**FIGURE 2.65   EFFECT OF FLAP OPERATION ON DRAG POLAR
(Ref. 7)**

2.9.4 Lift to Drag Ratio

The lift-drag ratio, L/D, is of importance in flight testing since it directly determines the glide, cruise and flame-out landing speeds and also affects other areas of performance. The lift-drag ratio can be determined directly from a drag polar since

$$\frac{L}{D} = \frac{C_L q_\infty S}{C_D q_\infty S} = \frac{C_L}{C_D}$$

Therefore, the slope of a line connecting the origin of the C_L versus C_D graph with any point on the polar is the lift-drag ratio as shown in Figure 2.66. The maximum L/D is the point of tangency of this line from the origin with the polar.

Aircraft lift-drag ratios are also quite often plotted versus lift coefficient or angle of attack as shown in Figure 2.67.
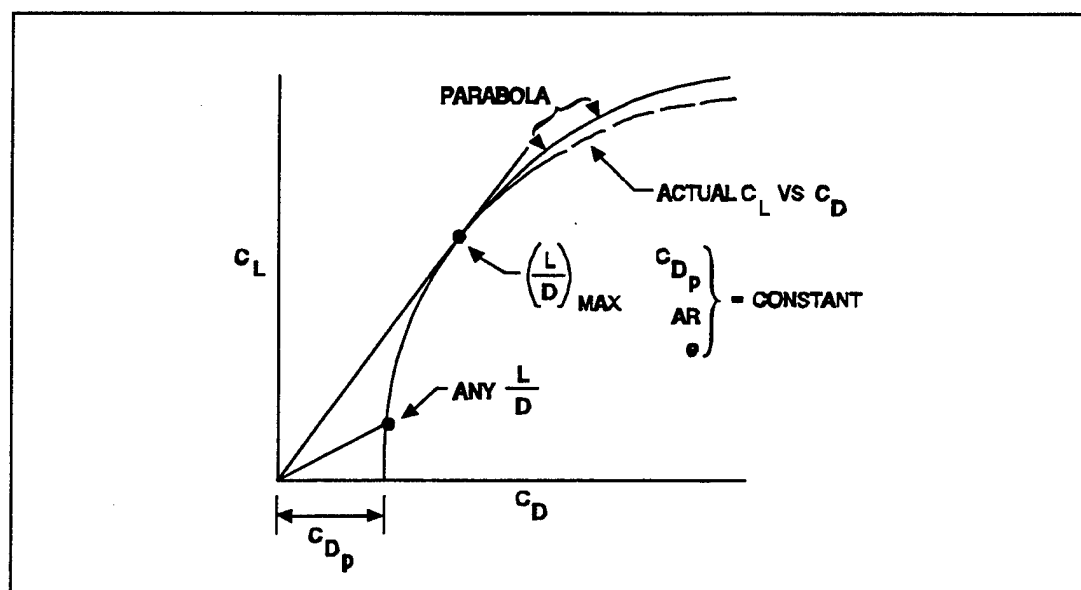
FIGURE 2.66   AIRCRAFT LIFT-DRAG RATIO
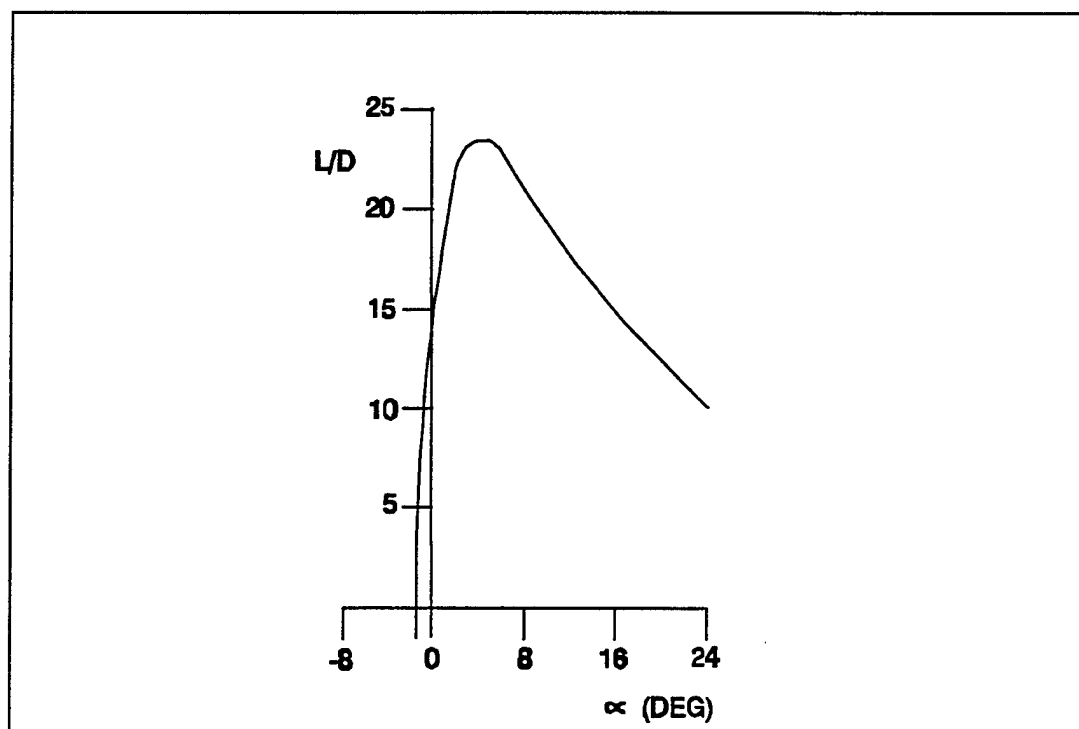
FIGURE 2.67   LIFT-DRAG RATIO FOR A WING OF ASPECT RATIO 6 WITH A NACA 23012 AIRFOIL SECTION AT A REYNOLDS NUMBER OF 8.37 X 10^6 (REF 3)

2.10 INTRO TO AIRCRAFT PERFORMANCE II

2.10.1 Equations of Motion

Consider the force diagram for an airplane in flight, as shown below:
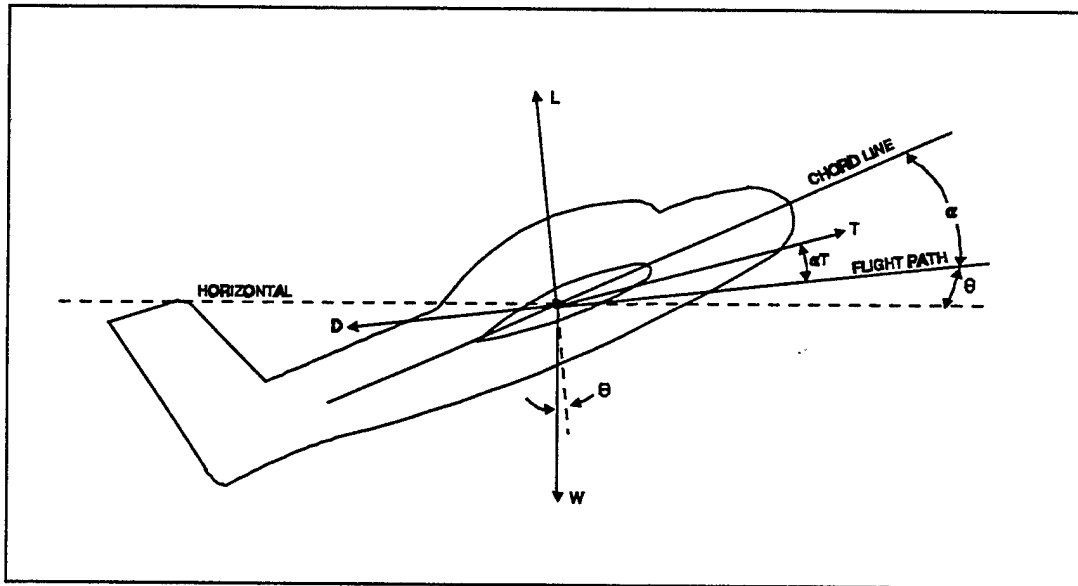
**FIGURE 2.68   FORCE DIAGRAM FOR AN AIRPLANE IN FLIGHT
(Ref. 1)**

By applying Newton's Second Law, both parallel and perpendicular to the flight path, the following are the equations of motion for an airplane in translational flight.

$$T\cos\alpha_T - D - W\sin\theta = m\frac{dV}{dt}$$

$$L + T\sin\alpha_T - W\cos\theta = m\frac{V^2}{r_c}$$

Let's now assume the following:

- No acceleration (so called "static performance")

$$\frac{dV}{dt} = 0 = \frac{V^2}{r_c}$$

- Straight flight

  r_c approaches infinity, therefore

$$\frac{V^2}{r_c} = 0$$

- Small thrust angle

  $\cos \alpha_T = 1$

  $\sin \alpha_T = 0$

- Level flight

  $\cos \theta = 1$

  $\sin \theta = 0$

With these assumptions, the equations of motion reduce to the following simple result:

$$L = W$$
$$T = D$$

In straight, level, and unaccelerated flight the aerodynamic drag is balanced by the thrust of the engine--the aerodynamic lift is balanced by the weight of the airplane.

2.10.2 Thrust Required for Level, Unaccelerated Flight

The thrust-required curve is a plot of the variation of thrust required (thrust necessary to overcome drag) versus velocity. Qualitatively, the curve looks like the following:
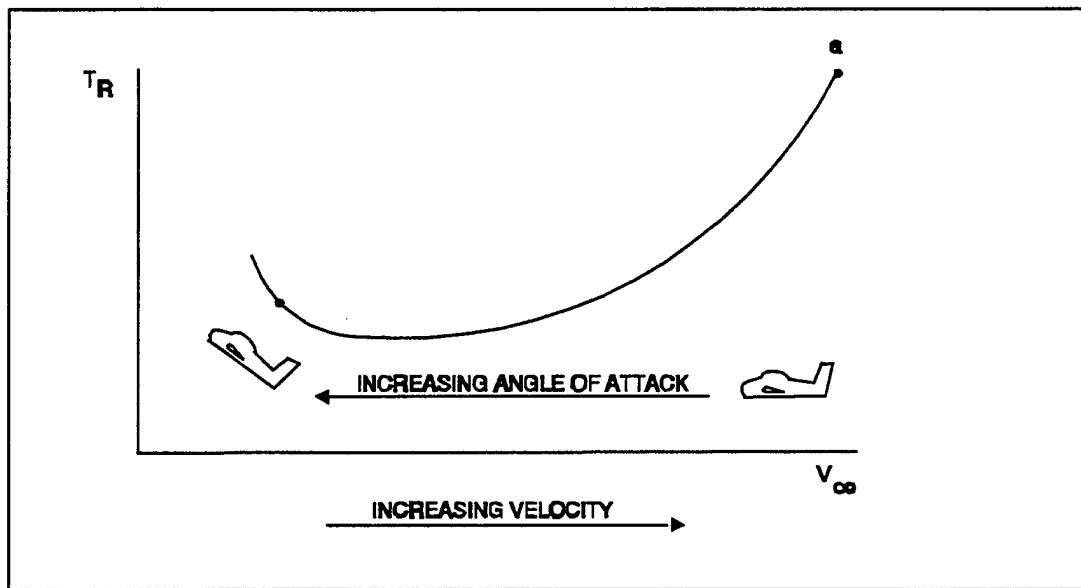
FIGURE 2.69   THRUST-REQUIRED CURVE WITH ASSOCIATED ANGLE-OF-ATTACK VARIATION (Ref. 1)

Recall that drag is composed of a parasite and an induced term (subsonic only)--a look at the drag equation is useful in understanding the shape of the thrust-required curve.

$$T_R = D = C_D q_\infty S = \left[C_{D,o} + K C_L^2 \right] q_\infty S$$

$$C_L = \frac{nW}{q_\infty S}$$

$$T_R = \left(C_{D,o} \frac{1}{2} \rho_\infty \right) V_\infty^2 + \left[\frac{K(nW)^2}{\frac{1}{2} \rho_\infty S} \right] \frac{1}{V_\infty^2}$$

From the equation, we see that at high airspeeds parasite drag dominates. At low speeds induced drag dominates...makes sense, right? Refer to the following figure.
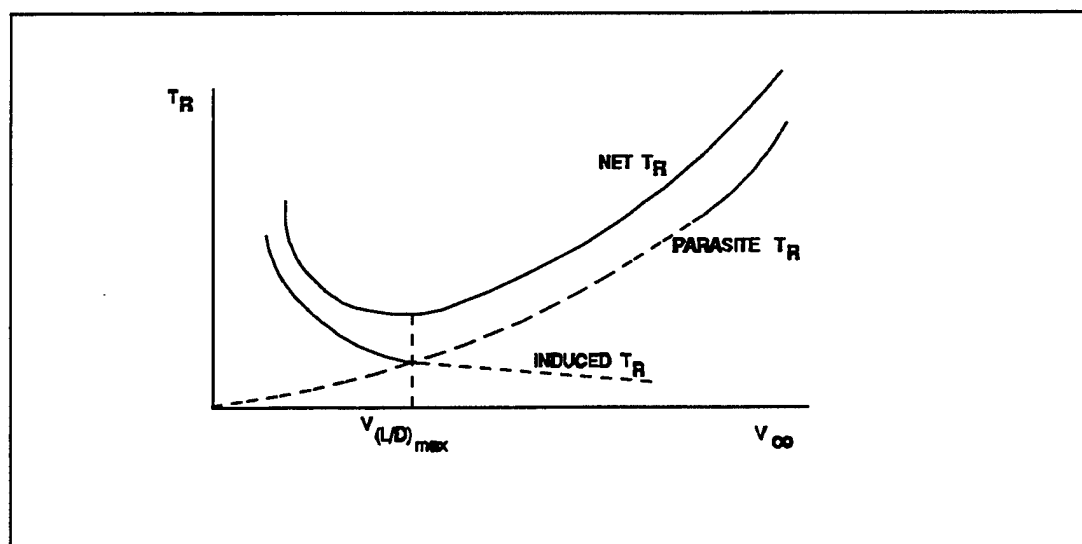
FIGURE 2.70   COMPARISON OF INDUCED AND PARASITE THRUST REQUIRED (Ref. 1)

If Mach effects are considered, then a drag rise due to compressibility begins at critical Mach, and drag versus Mach typically looks like Figure 2.71. Note in Figure 2.71 that Mach drag and total drag keep increasing supersonically, which must be the case since increasing values of thrust are required as speed increases supersonically.

An actual drag polar for an early supersonic fighter aircraft is shown in Figure 2.72 along with variations in drag coefficient with Mach for constant values of C_L.
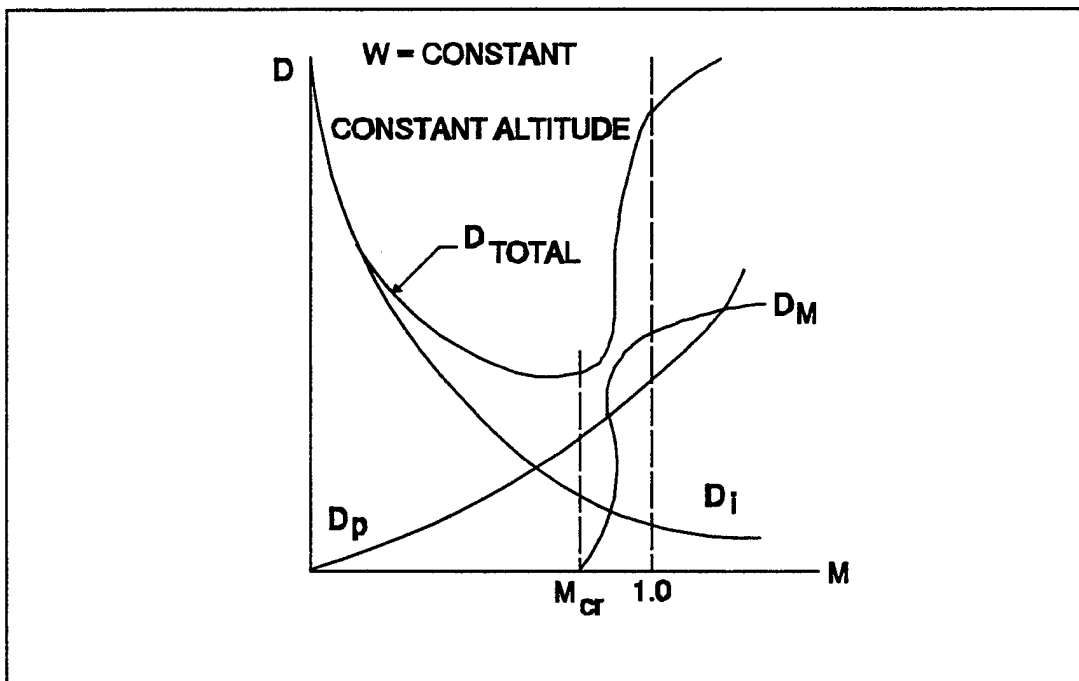
**FIGURE 2.71   VARIATION IN TOTAL DRAG WITH MACH FOR
STABILIZED LEVEL FLIGHT**
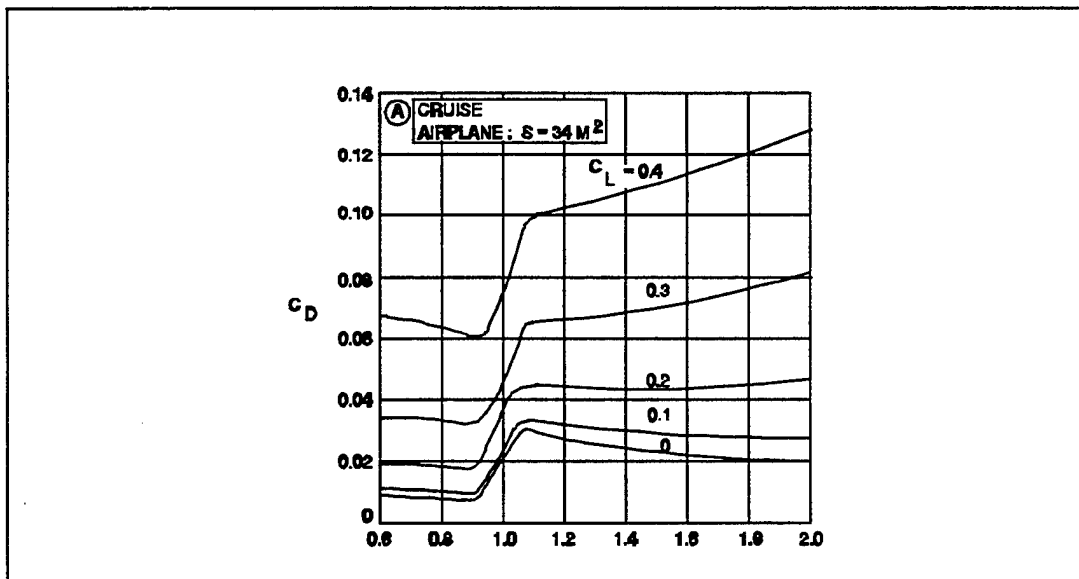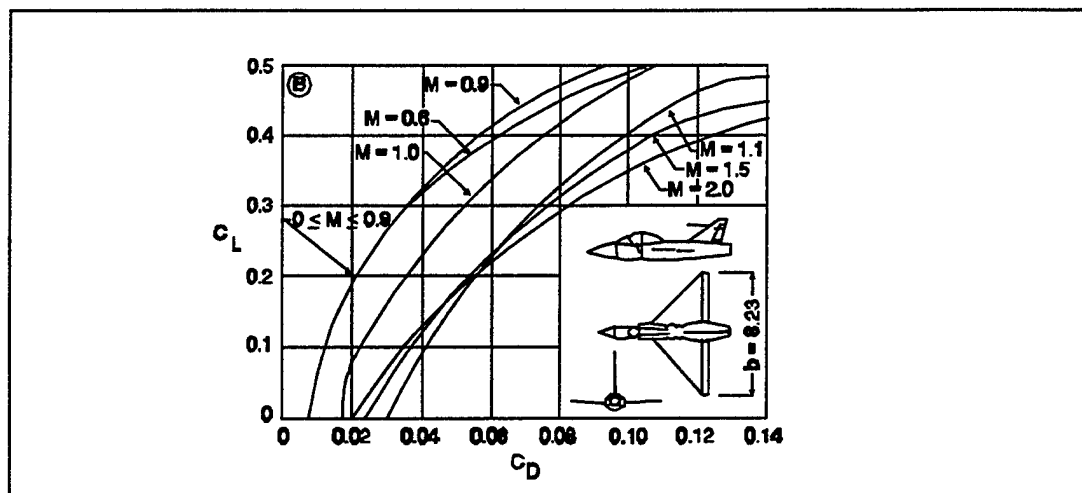
**FIGURE 2.72   DRAG CHARACTERISTICS OF AN EARLY SUPERSONIC
FIGHTER (Ref. 8)**

HOMEWORK PROBLEMS

1. An aircraft is flying at a geometric altitude, h_G, of 10,000 feet in air having a temperature of 500° Rankine and a pressure of 1,456 lb/ft². What is the density of the air?

Compare your result with the value given in the standard atmosphere tables, Appendix B. What might account for any difference?

2. An airplane is flying in still air at an altitude of 10 km and at a true airspeed of 300 knots.

   a. Give the speed of the airplane (V_{ac}) and speed of the earth (V_E) relative to

      1. an observer on the ground, V_{ac} = _____, V_E = _____.
      2. the pilot, V_{ac} = _____, V_E = _____

   b. Give the speed of the air (V_{air} far ahead of, and free from the airplane's disturbances) relative to

      1. an observer on the ground, V_{air} = _____
      2. the pilot, V_{air} = _____

3. The YF-16 has the following physical characteristics: wing span = b = 29.0 ft, wing planform area = S = 280 ft², design gross weight = 21,500 lb. Give the value of these characteristics and the design gross mass in S.I. (metric) units.

4. The Academy's Trisonic Wind Tunnel is operated from six large compressed air tanks. The tanks have a total volume of 153 m³ and are pumped up to a pressure of 4.137 x 10⁶ N/m² at 38°C. What is the mass of air contained in the tanks? Find the answer in kg and slugs.

5. What two mechanisms exist for transmitting an aerodynamic force to an object when air flows past it?

6. What is the weight, in Newtons, of a mass of 3 kg?

7. Complete the following table:

Quantity/Property	SI Units	English Units
time	s	s
length	m	ft
mass	kg	slug
force		
power		
density		
volume		
area		
specific volume		
pressure		
work/energy		

8. Explain the evolution of the NACA and NASA.

9. The Range Equation for constant altitude cruise is:

$$R = 2\left(\frac{2}{\rho S}\right)^{\frac{1}{2}} \left(\frac{1}{c}\right) \left(\frac{C_L^{\frac{1}{2}}}{C_D}\right)\left[W_0^{\frac{1}{2}} - W_1^{\frac{1}{2}}\right]$$

Find the cruise range, in nautical miles, of a T-38 at 30,000 feet altitude, given the following: (This is a typical units problem you will be expected to handle later in the course):

$S = 170$ ft^2 $c = .971/hr$ $C_L = .231$ $C_D = .022$ $W_0 = 11,000$ lb

$W_1 = 9,000$ lb $\rho = 8.907 \times 10^{-4}$ slug/ft^3 1 NM = 6,080 ft

10. An aircraft flying at a geometric altitude of 20,000 ft has instrument readings of $P = 900$ lb/ft^2 and $T = 460°$ Rankine. Find h_P, h_T, and h_ρ to the nearest 500 ft.

11. In your own words, why do aeronautical engineers and pilots need a standard atmosphere?

12. Using Appendix A of your text, what are the properties of the standard atmosphere at an altitude of 5500 m?

13. What are the properties of the standard atmosphere at 40,000 ft?

14. Define incompressible flow and give airspeeds that allow this assumption to be made for air.

15. Define steady flow and give an example of steady flow and one for non-steady flow.

16. A wind tunnel has the following flow properties at the inlet,
    $P = 101,000 \text{ N/m}^2$, $A = 1\text{m}^2$, $T = 288\text{K}$, $V = 200$ m/s

    and in the test section
    $A = 0.25\text{m}^2$ and $V = 900$ m/s. (Draw a sketch)

    a. What is the mass flow rate through the wind tunnel?

    b. What is the density in the test section?

c. Is the flow compressible or incompressible? Why?

17. Define one-dimensional flow and give an example.

18. Air flows in the inlet of a low speed jet with the following properties, P=14.5 psi, A=2.65 ft^2, V=300 ft/s, ρ=0.0024 slug/ft^3. At the end of the inlet is the compressor face where the pressure is 15 psi. Assume the flow is steady, incompressible, one-dimensional, and inviscid.
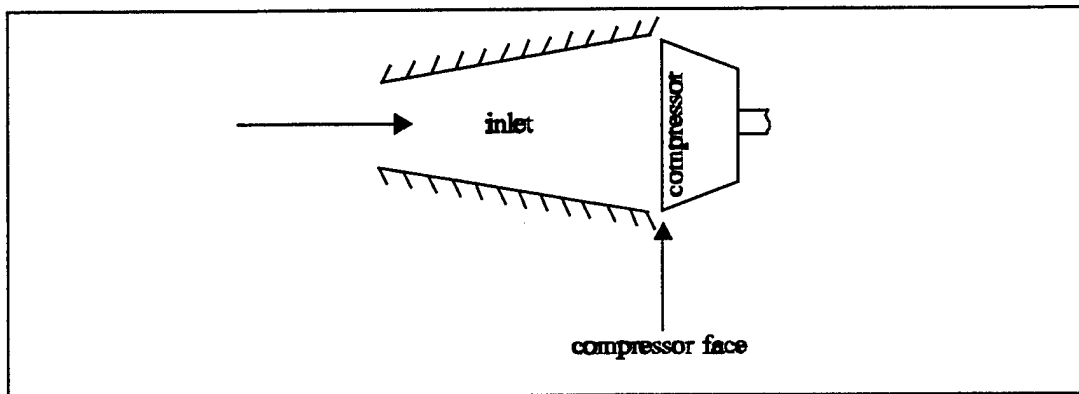
a. What is the mass flow rate?

    b. Determine the area at the compressor face.

19. Consider the steady flow of water through a rusty, rough pipe of constant cross-
sectional area. Some values of flow properties have been determined for two stations;
Station 1 is upstream of Station 2,

    $P_1 = 3000$ N/m^2, $V_1 = 3$ m/s, $A_1 = .01$ m^2, $\rho_1 = 1,000$ kg/m^3 and
    $P_2 = 2000$ N/m^2 and $\rho_2 = 1,000$ kg/m^3. Draw a sketch.

    a. Determine the velocity at Station 2.

    b. Calculate $(P + 1/2\rho V^2)$ at Stations 1 and 2. Are they equal?

    c. With reference to Bernoulli's Equation, explain whether the two values in
Part b should or should not be the same.

20. What were the major contributions of Bernoulli and Euler to the development of aerodynamics and fluid mechanics?

21. You are concerned about the moon roof on your new sports car. It seems to flex when driving at high speeds. Calculate how much net force the moon roof must withstand and in what direction. Assume the moon roof is flat with an area of 0.5 m^2 and the pressure and velocity over the moon roof is constant. Your driving speed is 20 m/s and the velocity over the moon roof is 30 m/s. The pressure inside the car is 90,500 N/m^2 and the freestream pressure and density in front of the car are 90,000 N/m^2 and 1.1 kg/m^3. Draw a sketch.

22. Assume standard day conditions for the following questions.

    a. What is the speed of sound at sea level in ft/s and m/s?

b. What is the speed of sound at 30,000 ft in ft/s and m/s?

c. What is the Mach Number of an aircraft flying at 600 ft/s at sea level?

d. If an aircraft's Mach Number is .7 at 30,000 ft what is its true velocity (in knots)?

23. An instrument used to measure the airspeed on many early low speed airplanes during the 1910-1930 time period, was the venturi, sketched below. This simple device is mounted at a specific location on the airplane where the inlet velocity is essentially the same as the free stream velocity. With a knowledge that $A_1/A_2 = 4$ and $P_1 - P_2 = 4000$ N/m², find the airplane's velocity at sea level.
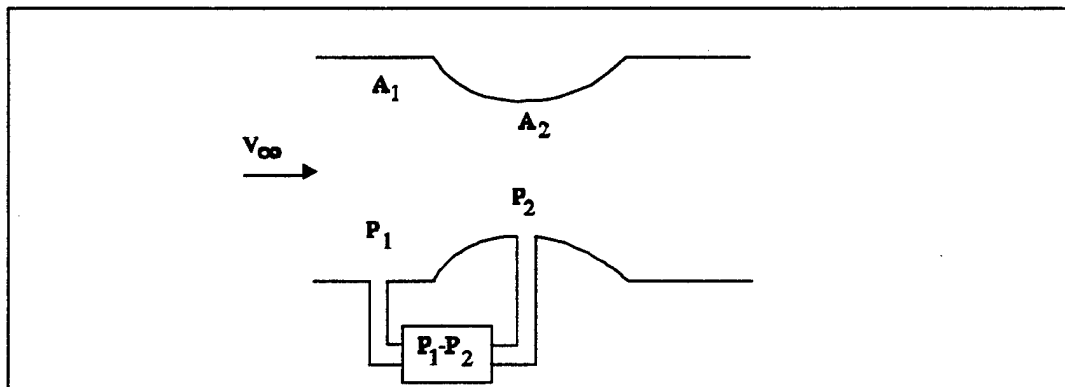
24. A new wing design is being tested at USAFA in low speed wind tunnel and in a flight test. Both tests take place in standard day conditions at an altitude of 2000 m. The velocity in the wind tunnel test section is 90 m/s and the true velocity of the flight test aircraft is 90 m/s. Draw a sketch.

        WIND TUNNEL   FLIGHT TEST

a. What are the atmospheric conditions for these tests in SI units?

b. Calculate the total pressure at the leading edge stagnation point for the airfoil in the wind tunnel. Assume no total pressure loss.

c. Calculate the total pressure at the leading edge stagnation point of the flight test wing. Hint: Use a reference attached to the aircraft.

25. The sketch below illustrates velocity profiles of the boundary layer at a few locations along a plate with distances in the y direction greatly exaggerated.
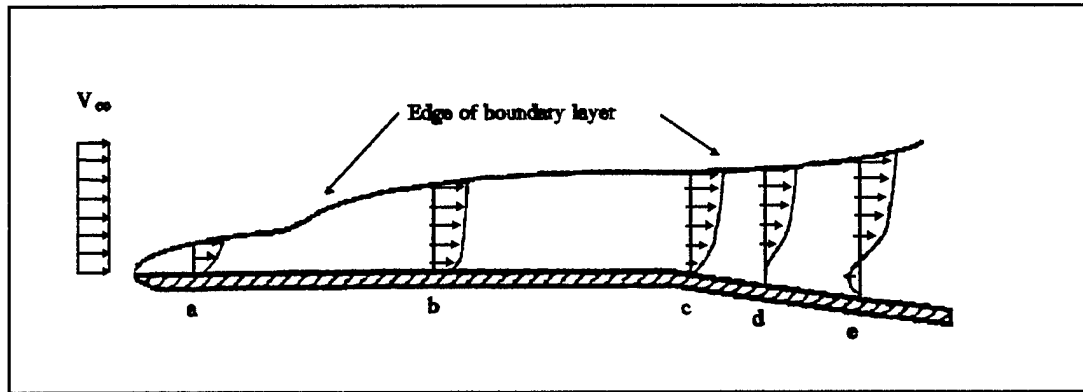
a. What type of boundary layer exists at point a?

b. What type of boundary layer exists at point b?

c. Transition occurs between what two profiles?

d. Reverse flow is evident in which profiles?

e. Where does separation occur?

f. Which direction would the transition point move if:
   (1) Pressure gradient is made more favorable.
   (2) Surface roughness is increased.
   (3) Freestream turbulence is reduced.

26. An aircraft is in flight at 10,000 ft standard day at M = 0.4. Its wing has a chord of 4 ft and a critical Reynolds number of 8×10^5.

    a. Find the overall or total Reynolds number for the wing.

    b. Find the distance back from the leading edge at which transition occurs.

    c. What is the boundary layer thickness one foot behind the leading edge, if the flow has a turbulent boundary layer from the leading edge?

27. The wing of the Fairchild Republic A-10 is approximately a rectangular shape with a span of 17.5m and a chord of 3m. If the A-10 is flying at sea level on a standard day at 100 m/s and we assume the wing is a flat plate:

     a. Calculate the boundary layer thickness at the trailing edge and the total skin friction drag assuming the flow is completely laminar.

     b. Calculate the same parameters assuming the flow is completely turbulent. Compare the laminar and turbulent results.

     c. If the critical Reynolds number for transition is 10^6, calculate where transition occurs.

     d. Using the critical Reynolds number in part c, what is the total skin friction drag?

28. Calculate the local Reynolds number at a point 2 ft aft of the leading edge of a wing being tested at sea level conditions and 800 fps. What type of boundary layer exists at this point? (Critical Re = 5 x 10^5.)

29.   a. What is an adverse pressure gradient and where does it occur on an airfoil?

       b. What causes the flow to separate from an airfoil?

       c. What are the two major consequences of flow separation?

       d. Why do golf balls have dimples?

30. The National Advisory Committee for Aeronautics has developed two series of low speed airfoils, designated the four-digit (such as 2212) and the five-digit (such as 23012) series.

The interpretation of the four digit series is as follows:

THE NACA FOUR-DIGIT SERIES

First digit - camber in percent chord

Second digit - position of maximum camber in tenths of chord from leading
edge

Last two digits - maximum thickness in percent chord

Example:

	NACA 2212	NACA 0009
maximum camber	0.02c	symmetrical
position of max camber	0.20c	no camber
maximum thickness	0.12c	0.09c

a. A NACA 2415 airfoil has a chord of 1.4m. Give the maximum chamber, its location, and the maximum thickness of the airfoil in units of meters.

b. A NACA 0015 airfoil has a 0.152m chord. Give the camber, its position, and the thickness of the airfoil in units of meters.

31. Draw a typical C_l vs α curve for a positively cambered airfoil and label the following features:

a. the axes (C_l vs α)

b. alpha at lift = 0

c. a_0

d. the stall angle of attack

e. C_l max

32. Consider a rectangular wing mounted in a low speed subsonic wind tunnel. The wing model completely spans the test section so that the flow sees essentially an infinite wing. The wing has a NACA 4412 airfoil section, a chord of 3.0 m and a span of 20 m. The tunnel is operated at the following test conditions: (See graph, page 136 in Introduction to Flight for viscosity)

$P = 101,000$ N/m² $T = 30°C$ $V = 48$ m/s

a. Determine the operating Reynolds number.

b. Calculate the lift, drag, and moment about the aerodynamic center for an angle of attack of 8° and $Re_L = 9 \times 10^6$.

    c. At a Reynolds number of 3×10^6:

       i) What is the stalling angle of attack for this airfoil?

       ii) What is the angle of attack for zero lift?

       iii) What is the lift curve slope?

33. Consider an NACA 2415 airfoil section in a low speed wind tunnel. $Re_L = 9 \times 10^6$

    a. What is the zero lift angle of attack?

    b. Does this airfoil have negative or positive camber?

    c. What is the stalling angle of attack?

    d. What is the maximum value of C_l?

    e. What is the lift curve slope?

34.   a. What is the aspect ratio (AR) of an A-10 with a wing span for 57 ft 6 in and a mean chord of 8 ft?

    b. An F-104 has a wing span of 21.9 ft and a wing area of 196 ft^2. What is the aspect ratio?

35.   a. What is the center of pressure (c.p.) of an airfoil?

    b. What is always true about the sum of aerodynamic force moments about the center of pressure?

36.   a. What is the aerodynamic center (a.c.) of an airfoil?

        b. A NACA 4412 airfoil has a chord of one meter. For $Re_L = 3 \times 10^6$, how far from the leading edge is the a.c. of this one meter chord airfoil? (see page 586 of Introduction to Flight) Does the location change with variation in Reynolds number?

37.   a. An airfoil at sea level conditions has a static pressure port on the top which measures a pressure of 98,000 N/m^2. Find the pressure coefficient at this point if the freestream velocity is 50 m/s.

        b. Consider the same airfoil except, instead of pressure, we measure velocity at the same point on top of the airfoil to be 89 m/s. Find Cp. (Hint: Derive an equation for Cp in terms of velocity using Eq. 5.23 of Introduction to Flight and Bernoulli's Eq. This equation is only accurate for incompressible flow.)

38.   a. A NACA 2415 airfoil (Introduction to Flight p. 584) has a lift coefficient of 0.6 at an angle of attack of 4 degrees. For what Mach numbers is this data accurate?

b. Estimate is the lift coefficient at M = 0.5?

39.   a. Define critical Mach number.

b. What two features can aircraft designers use to increase the critical Mach number of an aircraft?

40. A straight wing with a critical Mach number of .65 is sweptback 35°. What is its critical Mach number in the swept configuration?

41. What are two consequences when we have a wing (3 dimensional) instead of an airfoil (2 dimensional)?

42. Induced drag is also called due to _____. When lift is zero, the induced drag is _____.

43. Consider a flying wing made using a NACA 2412 airfoil with a wing area of 250 ft^2, a wing span of 50 ft, a span effectiveness factor of 0.9. If the aircraft is flying at a 6° angle of attack, and a Reynolds number of approximately 9 x 10^6, what is C_D for the flying wing?

44. When would induced drag be more prevalent; during high speed flight (such as cruise) or low speed flight (such a landing)? Why?

45. Derive the stall equation (Eq. 5.56 Introduction to Flight) from the basic lift equation (L = C_LqS).

46.   a. Draw a sketch of a typical C_L versus angle of attack curve and a C_D versus C_L curve and show how increasing camber (by putting a flap down) would change these curves.

        b. Draw the same curves and show how using boundary layer control (such as suction) would change these curves.

47. A flying wing with a chord of 12 ft is cruising at 400 ft/sec at 10,000 ft on a standard day. What is the flight Reynolds number based on wing chord length?

48. An F-15 is traveling at 700 KTAS at 60,000 ft on a standard day. What is the flight Mach number?

49. What true velocity (ft/sec and kts) is an SR-71 maintaining at Mach = 3.0 cruise at 65,000 ft on a standard day?

50. A T-38 flying on a standard day stabilizes at 37,000 ft at 96 percent RPM at Mach 0.80. Later at another point in the mission, while performing aerobatics, the pilot notices that at 45,000 ft with his power set at 96% RPM, he is accelerating through Mach 0.8.

    a. Compute the flight dynamic pressure for the two points described above.

    b. What is the equivalent airspeed at the 45,000 ft point described above?

51. An aircraft stabilizes at a pressure altitude of 30,000 ft in level unaccelerated flight. The ambient temperature at that level is measured to be -60°F. Is it a standard day? FIND: Ambient pressure in lb/ft², δ, θ, and σ.

52. A T-38 weighing 11,000 lb with a wing area of 170 ft² is stabilized in a 4-g turn at 20,000 ft on a standard day at an equivalent airspeed of 500 ft/sec. How much lift is the aircraft generating? What is the coefficient of lift (C_L)?

53. The RF-4C has a wing reference area of 530 ft². If its wingspan is 38.41 ft, what is its aspect ratio?

54. Given the lift curve shown below. Estimate the lift curve slope at a Mach of 0.7. Draw the estimated 0.7 Mach lift curve.
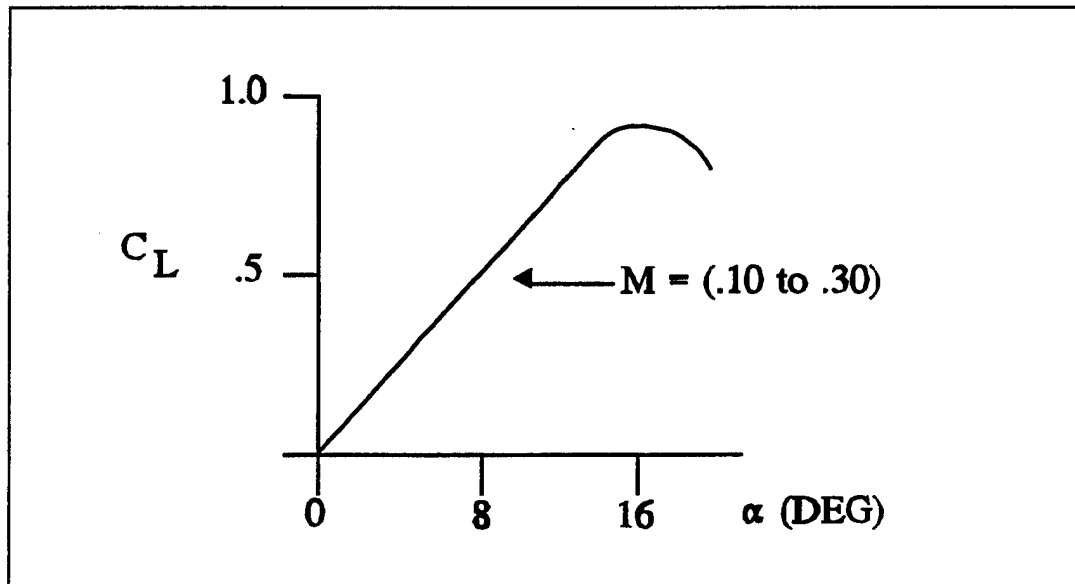
55. Find the pressure coefficient at Point P. The wing is traveling at 300 ft/sec on a standard day at sea level.
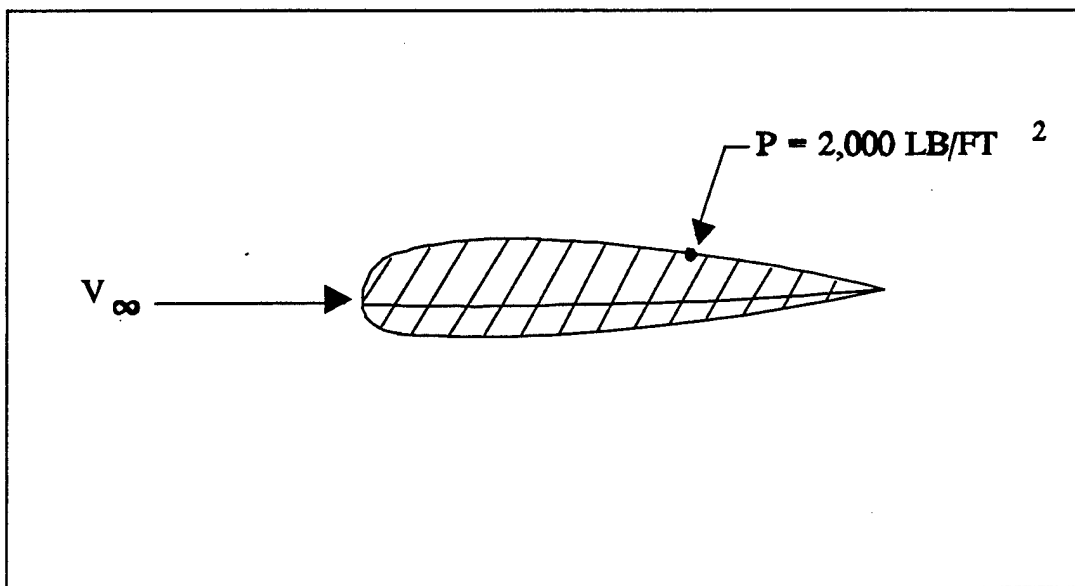
56. Plot changes in C_L vs α (symmetric airfoil) for:

    a. Increases in aspect ratio

    b. Increase in Reynolds number

    c. Extension of leading edge (LE) flaps

    d. Extension of trailing edge (TE) flaps

57. The wing of an aircraft operating at 10,000 ft on a standard day has a stagnation point where the pressure is 100 lb/ft^2 higher than atmospheric pressure and a maximum velocity point where the pressure is 200 lb/ft^2 lower than atmospheric. Assume that Bernoulli's equation applies.

    a. What is the free stream velocity in ft/sec?

    b. What is the maximum tangential velocity on the wing surface in ft/sec?

c. Where on the wing is the static pressure a minimum?

58. A T-38 is flying at 40,000 ft on a standard day at 500 ft/sec TAS. Assuming incompressible flow, what is the Bernoulli constant (total head) for this flight condition? What is the aircraft's flight Mach?

59. On a standard day an aircraft with a wing area of 691 ft^2 and a wingspan of 58.8 ft is in stabilized level flight cruising at an altitude of 38,300 ft and at a true velocity of 400 kts. The aircraft weighed 50,000 lb at takeoff and has since burned 10,000 lb of fuel. The SAC pilot also consumed his three lb lunch immediately after takeoff. The ten engines on the aircraft are producing 300 lbs of thrust each.

a. What are the aircraft lift and drag coefficients for these flight conditions?

   b.   The same aircraft in preparation for landing has slowed down to an airspeed of 150 ft/sec at a gross weight of 18,478 lb in cruise configuration and has stabilized in level flight. Each engine is now producing 166.3 lbs of thrust. What are the aircraft lift and drag coefficients for these flight conditions?

60. Shown below is the drag polar for the YX-007 in the power approach configuration obtained from flight test. The YX-007 has a wingspan of 40 ft and a wing area of 400 ft^2. Determine:
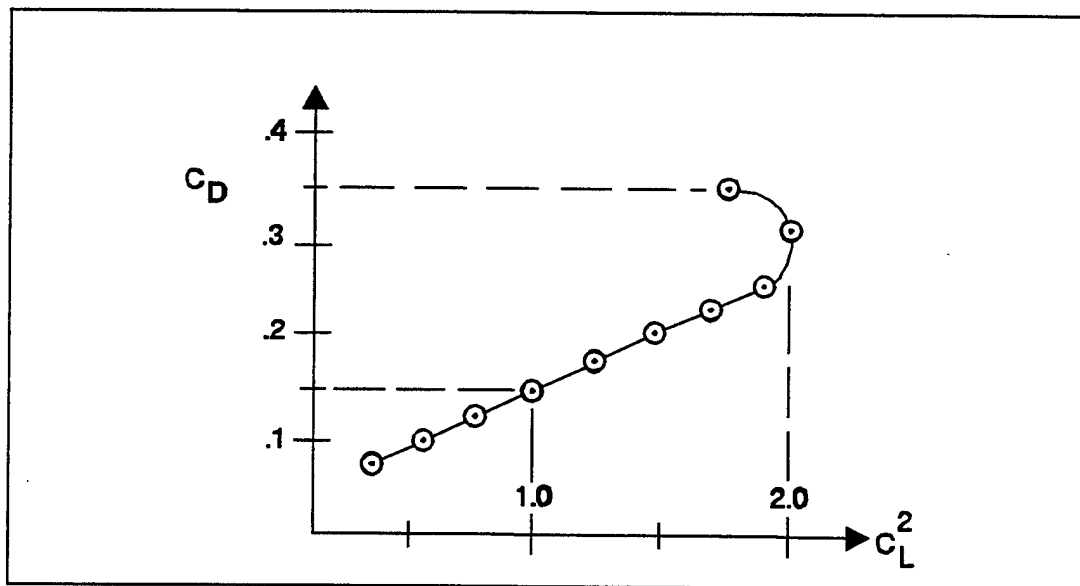
a. The aircraft's aspect ratio.

b. The aircraft's parasite drag coefficient.

c. The aircraft's efficiency factor(e).

d. The $C_{D_{total}}$ equation for the aircraft as a function of the lift coefficient.

e. If the final approach is to be flown at 80% of maximum lift coefficient, what is the final approach equivalent airspeed if aircraft weight = 12,100 lb?

f. The final approach L/D ratio if the approach is flown at 80% of maximum lift coefficient

g. The aircraft's equivalent flat plate area in the power approach configuration given that the drag coefficient of a flat plate is 1.28.

61. Given the following drag polar, determine:
    AIRCRAFT DATA: Wingspan = 60 ft
                     Wing Area = 900 ft^2
                     Cruise Configuration
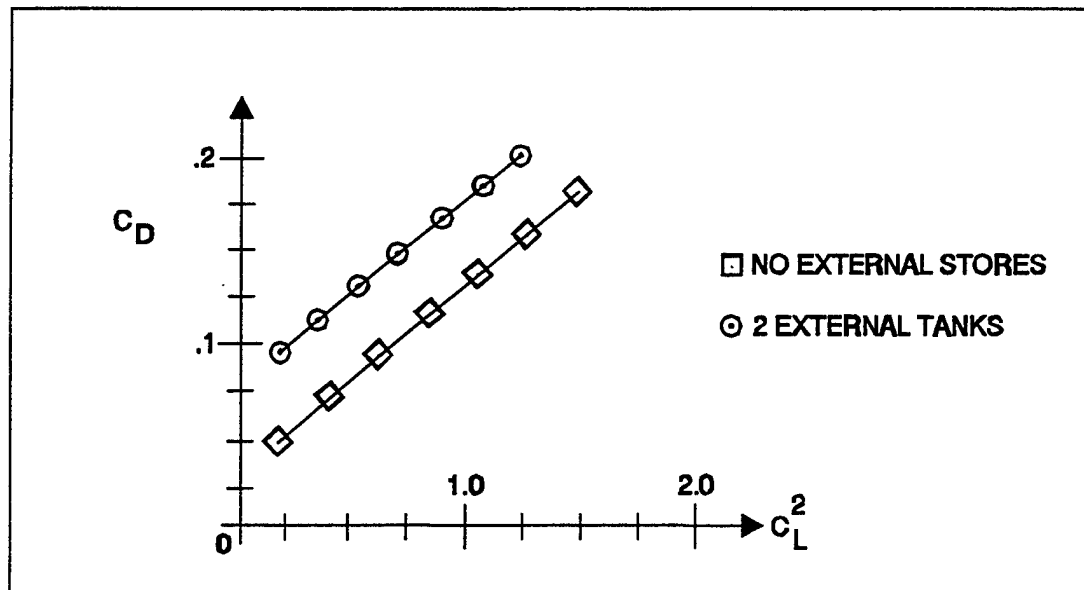
a. Aircraft's parasite drag with 2 external tanks installed.

b. The drag count of the 2 external stores.

c. The aircraft's maximum L/D with the 2 external stores.

62. For the aircraft represented by the plot below, find maximum lift-drag ratio, and parasite drag coefficient.
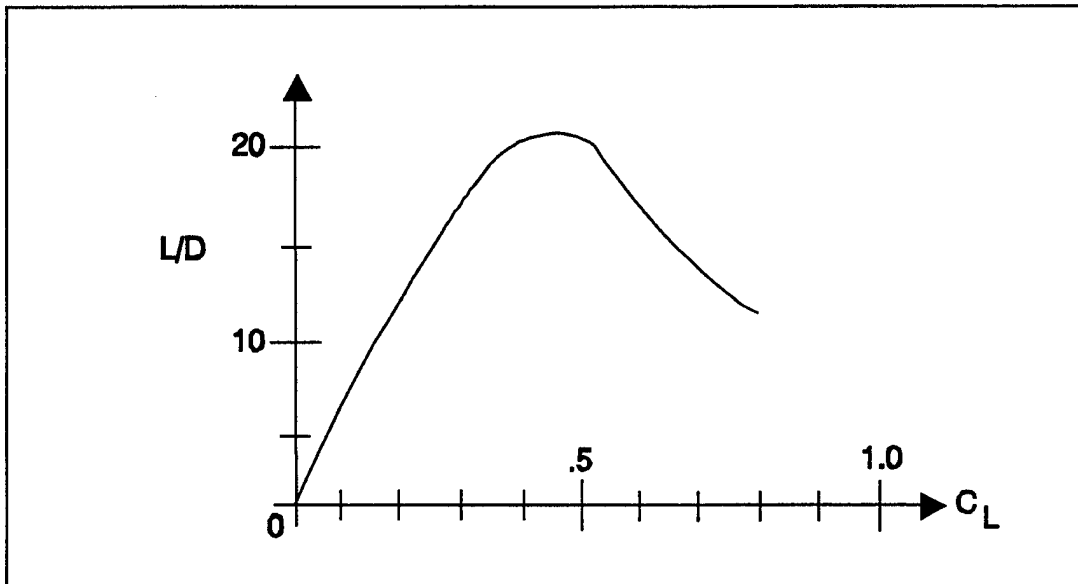

63. On a standard day an aircraft with a wing area of 691 ft^2 and a wingspan of 58.8 ft is in stabilized level flight cruising at an altitude of 38,300 ft and at a true velocity of 400 kts. The aircraft weighed 50,000 lbs at takeoff and has since burned 10,000 lbs of fuel. The ten engines on the aircraft are producing 300 lbs of thrust each.

    a. What are the aircraft lift and drag coefficients for these flight conditions?

b.   The same aircraft in preparation for landing has slowed down to an airspeed of 150 ft/sec (V_0) at a gross weight of 18,478 lb in cruise configuration and has stabilized in level flight. Each engine is now producing 166.3 lbs of thrust. What are the aircraft lift and drag coefficients for these flight conditions?

c.   Write the $C_{D_{total}}$ equation for the aircraft as a function of lift coefficient.

d.   Compute induced, parasite and total drag for the aircraft in Part A.

    e. Write the total drag coefficient equations for the aircraft described as a function of weight and Mach or equivalent velocity. What is the advantage of the equivalent velocity equation?

64. Two 500 lb bombs with a total drag count of 15 are installed on an A-37. The drag coefficient equation of the A-37 with the bombs installed is given below. The aircraft wing area is 184 ft^2.

$$C_{D_{total}} = 0.0235 + 0.062C_{L}^{2}$$

    a. What is the drag coefficient equation of the A-37 after the bombs are removed?

    b. If the aspect ratio of the A-37 is 6.2, what is the Oswald efficiency factor with the bombs installed? What is it after they are removed?

c. The same bombs are to be installed on a AF-6 whose wing area is 120 ft². The drag coefficient equation of the AF-6 is given below. What is the new drag coefficient equation with the bombs installed?

$$C_{D_{total}} = 0.03 + 0.062C_L^2$$

d. What was the equivalent flat plate area of the AF-6 before the bombs were installed? What is it after they are installed?

Answers

1. .001697 slugs/ft^3
2. a. (1) 300 kts, 0 kts   (2) 0 kts, 300 kts
   b. (1) 0 kts   (2) 300 kts
3. b = 8.84m, S = 26m^2, m = 9744 kg
4. m = 486 slugs,   m = 7091kg
5. Shear and Pressure forces
6. W = 29.4N
9. R = 969.5 NM
10. h_p = 22,000 ft, h_t = 16,500 ft, h_ρ = 23,000 ft
12. T = 252.4K, P = 5.054 X 10^4 N/m^2, ρ = .6975 kg/m^3
13. T = 390°R, P = 393.1 lb/ft^2, ρ = 5.873 x 10^{-4} slugs/ft^3
16. a. m = 244 kg/s   b. 1.08 kg/m^3 c. Compressible (V>100m/s)
18. a. m=1.91 slugs/s   b. $A_{comp.face}$ = 4.59 ft^2
19. a. $V_1 = V_2$ = 3 m/s   b. P_{0_1} = 7500 N/m^2 P_{0_2} = 6500 N/m^2
21. Force = 387.5N
22. a. a = 1116 ft/s, a = 340 m/s   b. a = 995 ft/s, a = 303 m/s
    c. M = .538   d. V = 412 kts
23. V_1 = 20.9 m/s
24. a. T = 275.16K, P = 79501 N/m^2, ρ = 1.0066 kg/m^3
    b. P = 79501 N/m^2 c. P_0 = 83,578 N/m^2
26. a. Re_L = 8.57 x 10^6 b. x_{cr} = .374 ft   c. δ_{turb} = .02 ft
27. a. δ_{lam} = .0034m, D_f = 188.4N   b. δ_{turb} = .0383m, D_f = 1640N
    c. x_{cr} = .146m   d. D_f = 1536N
28. Re_x = 1.02 x 10^7
30. a. Max Camber = .028m, Location = .56m aft, Max Thickness = .21m
    b. Max Thickness = .0228m
32. a. Re = 9 x 10^6 b. L = 100,200 N/,   D = 800 N/,
    M = -24,040 N-m
    (above answers may vary slightly due to differences reading the charts)
    c. 13 deg   d. -3.8 deg   e. .1/deg or 5.73/rad
33. a. -2 deg   b. positive   c. 16 deg   d. $C_\ell$max = 1.64
    e. .1/deg or 5.73/rad
34. a. AR = 7.2   b. AR = 2.45
37. a. C_p = -2.17   b. C_p = -2.17

38. a. Incompressible Mach numbers (M<0.3)   b. C_ℓ = .69

40. M_{cr}, swept = .794

42. Lift, zero

43. C_D = .023

47. 2.4×10^7 (No Units)

48. 1.22 (No units)

49. 1,718 kts; 2,905 ft/sec

50. a. 203 lb/ft^2, 138 lb/ft^2 b. 341 ft/sec

51. θ = 0.771, δ = 0.297, σ = 0.385 (No units)

52. 44,000 lbs; 0.87 (No units)

53. 2.78 (No units)

55. -1.084 (No units)

57. a. 338 ft/sec   b. 585 ft/sec

58. 465 lb/ft^2; M = 0.52 (No units)

59. a. 0.40, 0.03 (No units)   b. 1.00, 0.09 (No units)

60. a. AR = 4.0

    b. C_{D_P} = 0.05

    c. e = 0.8

    d. C_D = .05 + .10 C_L^2

    e. V_e = 150 ft/sec

    f. L/D = 6.35

    g. S_{plate} = 15.63 ft^2

61. a. C_{D_P} = 0.075

    b. 500 counts

    c. $(L/D)_{max}$ = 5.77

62. $(L/D)_{max}$ = 20 C_{D_P} = 0.01

63. a. 0.40, 0.03 (No units)

    b. 1.00, 0.09 (No units)

    d. 1,137 lb, 1,901 lb,   3,000 lb

64. a. C_D = .0220 + .062 C_L^2

    b. 0.828 (No units)

    d. 2.81 ft^2,   3.03 ft^2

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